Dynamic Price Competition, Learning-By-Doing and Strategic Buyers

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July 19, 2021

Abstract

We examine how strategic buyer behavior affects equilibrium outcomes in a model of dynamic price competition where sellers benefit from learning-by-doing by allowing each buyer to expect to capture a share of future buyer surplus. Many equilibria that exist when buyers consider only their immediate payoffs are eliminated when buyers expect to capture even a modest share of future surplus, and the equilibria that survive are those where long-run market competition is more likely to be preserved. Our results are relevant for antitrust policy and our approach may be useful for future analyses of dynamic competition.

Keywords: dynamic competition, learning-by-doing, strategic buyers, dynamic games, multiple equilibria.

JEL codes: C73, D21, D43, L13, L41.

*Corresponding author: atsweet@umd.edu. We are very grateful to Uli Doraszelski and Steve Kryukov for access to their computational results which helped to confirm our own results with non-strategic buyers. We are also very grateful to the co-Editor, Liran Einav, three referees, David Besanko and Uli Doraszelski for insightful comments that improved the content and presentation of the paper, as well as to conference and seminar participants at Penn State, the Federal Trade Commission and the Washington DC IO conference. Xinlu Yao's work on the project was completed while she was a PhD student at the University of Maryland. All errors are our own.

1 Introduction

In many industries, producers' marginal costs tend to fall with their accumulated past output (learning-by-doing, LBD).[1] LBD creates a tension between achieving productive efficiency by concentrating production at a single producer, and sustaining meaningful competition, which may require spreading production across multiple producers. A range of dynamic models has been used to study whether, without regulation, market competition is likely to be sustained and whether outcomes are likely to be efficient.

Two literatures making different assumptions about buyers have reached qualitatively different conclusions. For example, Lewis and Yildirim (2002) (LY) consider a model where, in each period, two suppliers that benefit from LBD compete to sell a single unit to a long-lived, forward-looking monopsonist. LY's model has a unique Markov Perfect equilibrium where the monopsonist spreads its purchases between the suppliers to maintain competition, even though this has the effect of raising prices.[2] On the other hand, some well-known models predict that a single seller may come to dominate the market when buyers are assumed to be atomistic. For example, Cabral and Riordan (1994) (CR) consider a model where duopolists sell differentiated products, LBD stops once a certain level of cumulative sales is reached, and there is an infinite sequence of short-lived buyers with idiosyncratic preferences over the sellers.[3] CR show, using an example, that if it is possible for sellers to exit, equilibria exist where the market may become a monopoly after initially intense competition. Besanko, Doraszelski, and Kryukov (2014) (BDK1) and Besanko, Doraszelski, and Kryukov (2019) (BDK2) show, using a richer version of CR's model (the BDK model), that this type of equilibrium exists for a broad range of parameters, and that these equilibria often co-exist with equilibria where duopoly will be sustained and initial pricing is less aggressive.

In practice, many industries with LBD, while not being monopsonies, have large, repeat

[1] Industries with documented LBD include airframes (Alchian (1963), Benkard (2000)), chemicals (Lieberman (1984), Lieberman (1987)), semiconductors (Irwin and Klenow (1994),Dick (1991)), shipbuilding (Thompson (2001), Thornton and Thompson (2001)), power plant construction (Zimmerman (1982), Joskow and Rose (1985)) and hospital procedures (Gaynor, Seider, and Vogt (2005), Dafny (2005)).

[2] The monopsonist also seeks to maintain competition in the related models of Lewis and Yildirim (2005) and Anton, Biglaiser, and Vettas (2014).

[3] CR also assume that sellers' costs are observed and that buyers' idiosyncratic preferences over sellers are private information, whereas LY assume that each seller's marginal cost contains an element that is idiosyncratic and private information.

buyers who are likely both to care about future competition and to recognize that their purchase choices may affect how competition evolves.[4] In this article, we extend the BDK model in a tractable way to cover these intermediate cases. Specifically, we model buyers who expect to capture a particular share of future buyer surplus, and therefore partially internalize how their purchase choices affect how the market evolves.

We analyze how equilibrium outcomes and welfare vary with this measure of how "strategically" buyers behave. Atomistic buyers and monopsony are polar cases. We find that the multiplicity of equilibria is eliminated across a broad range of the parameter space as buyers become more strategic. The equilibria that survive have a higher probability, or certainty, of sustained long-run competition, and they tend to increase total surplus even though a softening of competition may leave buyers worse off. While these qualitative results are unsurprising given LY's results, a novel finding is that we observe these changes even when the degree of strategic behavior is fairly low. For example, for the parameters that BDK1 use as their leading example, there is a unique equilibrium with permanent duopoly once each buyer expects to capture 15% of future buyer surplus.

Our method and our results make several contributions. First, allegations of anticompetitive conduct often come from industries where LBD, network effects or switching costs can lead to an incumbent's high current market share creating a lasting competitive advantage. The decision to initiate an investigation will often turn on whether the agency determines that features of the industry, including the sophistication of buyers (who may be large distributors, rather than final customers), are plausibly consistent with an exclusionary equilibrium.[5] While we consider a specific model of LBD that is not designed to capture the features of any particular market, our results suggest that a theory of inefficient exclusion may be less plausible when buyers are likely to be even moderately strategic.[6]

[4] Even government procurement may not be a monopsony if different agencies or governments in different juristictions purchase from the same suppliers.

[5] The Department of Justice's report "Competition and Monopoly: Single-Firm Conduct Under Section 2 of the Sherman Act" (`https://www.justice.gov/atr/competition-and-monopoly-single-firm-conduct-under-section-2-sherman-act`) discusses the issues involved in challenging allegedly anticompetitive conduct, although it was withdrawn as official policy in 2009.

[6] While BDK do not advocate for any particular anti-predation screen or policy, they use their results to suggest that predation is a real phenomenon that agencies should invest in trying to prevent. For example, BDK1 (p. 892): "Our analysis suggests that guiding these expectations toward "good" equilibria by creating

Second, we believe that our tractable formulation of how strategically buyers behave may be usefully applied to models where dynamics arise from other sources, such as network effects, or product durability or perishability. Existing models with strategic buyers (e.g., Gul, Sonnenschein, and Wilson (1986), Besanko and Winston (1990), Levin, McGill, and Nediak (2010), Jerath, Netessine, and Veeraraghavan (2010), Hörner and Samuelson (2011), Board and Skrzypacz (2016), Chen, Farias, and Trichakis (2019) for models with a monopolist seller, and under oligopoly, Levin, McGill, and Nediak (2009)) allow buyers to choose when to buy and do not consider what happens as the degree to which buyers are strategic varies. In contrast, we vary buyer strategicness in a setting where buyers can influence future market structure. Our formulation may also be useful in extending the empirical literature on estimating games with dynamic competition (e.g., Benkard (2004) and Kim (2014) estimate games where sellers benefiting from LBD are dynamic but buyers are static).

Third, we make a methodological contribution with a new algorithm to identify equilibria. Following BDK, we use homotopies as our primary method for identifying Markov equilibria. However, homotopies are not guaranteed to find all equilibria, so we also use a new recursive algorithm that, under some plausible assumptions, can identify whether a particular type of equilibrium that may result in monopoly exists. While backwards recursion is widely used to solve finite horizon sequential games, or games that must end up in a single absorbing state (for example, CR's model when there is no exit), we believe that we are the first to use it to test whether or not a particular type of equilibrium exists.[7]

The rest of the paper proceeds as follows. Section 2 describes the extended version of BDK's model. Section 3 explains why strategic buyer behavior changes equilibrium outcomes

a business environment in which firms anticipate that predatory pricing "does not work" (by issuing general guidelines about how allegations of predation are handled, speaking out against predation, pursuing high-profile cases, etc.) can be a powerful tool for antitrust policy." and p. 894: "Behavior resembling conventional notions of predatory pricing - aggressive pricing followed by reduced competition - arises routinely. This casts doubt on the notion that predatory pricing is a myth and does not have to be taken seriously by antitrust authorities." We agree with both of these statements, and view our work as highlighting that, in assessing alleged predation, it may be more important than previously recognized to account for how strategically buyers behave.

[7]We thank a referee for pointing out the novelty of our approach. Iskhakov, Rust, and Schjerning (2016) show that recursive algorithms can be used to identify equilibria in stochastic Markov equilibrium games where all movements through the state space must satisfy a directional property. One can view our approach as using recursion to identify the existence of a specific type of equilibrium where movements through parts of the state space are directional.

using BDK's baseline parameters. Section 4 shows that we see qualitatively similar patterns for different degrees of LBD and different degrees of product differentiation. Sections 3 and 4 find equilibria using homotopies. Section 5 provides supporting evidence about how strategic buyer behavior affects the types of equilibria that exist using our recursive algorithm, and examines the robustness of our results to changing several of the model's assumptions. Section 6 concludes. The Online Appendices contain details of the methods used, as well as additional figures and results.

2 Model

In this section we briefly describe the model. BDK1 and BDK2 provide additional motivation.

Overview. Two ex-ante symmetric but differentiated sellers and a set of symmetric strategic buyers play an infinite horizon, discrete time, discrete state dynamic game. Each seller i has a publicly observed state variable e_i, and is either a potential entrant ($e_i = 0$), or active with $e_i \in \{1, 2, ..., M\}$, which represents the seller's "know-how". Every period, active sellers set prices to compete for the unit demand of a buyer. An active seller's marginal cost is $\kappa\rho^{\log_2(\min(e_i,m))}$ where $\rho \in [0, 1]$ is the "progress ratio". For states below m, a doubling of know-how implies a $100(1-\rho)\%$ marginal cost reduction, but there is no marginal cost reduction when know-how increases above m. Marginal costs are constant for $e_i \geq m$, and $e_i \leq M$ constrains the state space to be finite. We follow BDK in assuming $\kappa = 10$, $M = 30$, $m = 15$ and a discount factor of $\beta = \frac{1}{1.05}$.

A buyer's flow indirect utility if it buys from seller i is $v_i - p_i + \sigma\epsilon_i$, where $v_1 = v_2 = 10$, p_i is i's price, and σ parameterizes the degree of product differentiation. The no purchase option (0) has $v_0 - p_0 = 0$. We model strategic buyers by assuming that the chosen buyer in each period is drawn, with replacement, from a pool of symmetric potential buyers, and that each buyer expects to be the buyer in any future period with probability $0 \leq b^p \leq 1$. The ϵ_is are private information Type I extreme value payoff shocks which are i.i.d. across buyers, options and periods, and do not depend on a buyer's past purchases. Buyers and

Figure 1: Within-Period Timing

a. Buyer selected from pool. Its tastes are private information.	b. Active sellers simultaneously set prices.	c. Buyer makes purchase choice.	d. Successful seller's experience increases by 1 (up to M).	e. Private info. seller entry costs and scrap values revealed and simultaneous entry/exit choices.	f. State space evolves given entry/exit choices.

sellers cannot sign multi-period contracts, and we ignore the effects that possible donwstream competition between buyers may have on purchase behavior.

Timing, State Transitions and Entry/Exit. Figure 1 summarizes within-period timing. Active sellers simultaneously set prices, without knowing the buyer's ϵs. A sale raises a seller's state by 1, unless it is already at M. There is no know-how depreciation. Sellers make simultaneous exit and entry choices. Sunk entry costs and scrap values are drawn independently from symmetric triangular distributions, with CDFs F_{enter} and F_{scrap}, and supports $[\overline{S}-\Delta_S, \overline{S}+\Delta_S]$ and $[\overline{X}-\Delta_X, \overline{X}+\Delta_X]$, respectively, with $\Delta_X, \Delta_S > 0$. The finite supports mean that entry may be certain or never optimal, and that exit may never be optimal.[8] We will use BDK's baseline parameter values, $\overline{S} = 4.5$, $\overline{X} = 1.5$, $\Delta_S = \Delta_X = 1.5$.

Equilibrium. We consider symmetric and stationary Markov Perfect Nash equilibria (MPE, Ericson and Pakes (1995), Maskin and Tirole (2001)). Existence of at least one MPE follows from Doraszelski and Satterthwaite (2010). An equilibrium will consist of, for each state $\mathbf{e} = (e_1, e_2)$, active seller prices ($p(\mathbf{e})$) and seller continuation probabilities ($\lambda(\mathbf{e})$), and the values of the sellers and a representative buyer in the pool defined at the start of the period ($V^S(\mathbf{e})$, $V^B(\mathbf{e})$) and before private entry/exit decisions are taken ($V^{S,INT}(\mathbf{e})$, $V^{B,INT}(\mathbf{e})$). Assuming, for simplicity, that $\sigma = 1$, equilibrium values and strategies will solve the following equations, where symmetry implies that we can express the equations in terms of seller

[8] When the support of the scrap value is wide enough, a seller that draws a low scrap value will always prefer to remain in the market.

1's strategies and values only.[9,10]

Beginning of period value for seller 1 (V_1^S):

$$V_1^S(\mathbf{e}) - D_1(p(\mathbf{e}), \mathbf{e})(p_1(\mathbf{e}) - c_1(e_1)) - \sum_{k=0,1,2} D_k(p(\mathbf{e}), \mathbf{e}) V_1^{S,INT}(\mathbf{e}'_k) = 0, \tag{1}$$

where $\mathbf{e}'_1 = (\min(e_1+1, M), e_2)$, $\mathbf{e}'_2 = (e_1, \min(e_2+1, M))$ and $\mathbf{e}'_0 = (e_1, e_2)$, i.e., the states that the game will transition to if there is a purchase from seller 1 or seller 2, or no purchase, respectively. The sale probabilities, D_k, will be defined below.

Value for seller 1 before entry/exit stage ($V_1^{S,INT}$) :

$$V_1^{S,INT}(\mathbf{e}) - \left(\begin{array}{c} \beta\lambda_1(\mathbf{e})\lambda_2(\mathbf{e})V_1^S(\mathbf{e}) + \beta\lambda_1(\mathbf{e})(1-\lambda_2(\mathbf{e}))V_1^S(e_1, 0) + \\ (1-\lambda_1(\mathbf{e}))E(X|\lambda_1(\mathbf{e})) \end{array} \right) = 0, \tag{2}$$

for $\mathbf{e} = (e_1, e_2)$ where $e_1, e_2 > 0$, with similar equations when a seller is a potential entrant. $E(X|\lambda_1(\mathbf{e}))$ is the expected scrap value when seller 1 chooses to exit.

First-order condition for seller 1's price (p_1) if $e_1 > 0$:

$$D_1(p(\mathbf{e}), \mathbf{e}) + \sum_{k=0,1,2} \frac{\partial D_k(p(\mathbf{e}), \mathbf{e})}{\partial p_1} V_1^{S,INT}(\mathbf{e}'_k) + (p_1(\mathbf{e}) - c_1(e_1)) \frac{\partial D_1(p(\mathbf{e}), \mathbf{e})}{\partial p_1} = 0 \tag{3}$$

Seller 1's continuation probability in entry/exit stage (λ_1):

$$\lambda_1(\mathbf{e}) - F_{\text{enter}}(\beta\left[\lambda_2(\mathbf{e})V_1(1, \max(1, e_2)) + (1-\lambda_2(\mathbf{e}))V_1(1, 0)\right]) = 0 \text{ if } e_1 = 0, \tag{4}$$

$$\lambda_1(\mathbf{e}) - F_{\text{scrap}}(\beta\left[\lambda_2(\mathbf{e})V_1(e_1, \max(1, e_2)) + (1-\lambda_2(\mathbf{e}))V_1(e_1, 0)\right]) = 0 \text{ if } e_1 > 0. \tag{5}$$

[9] For example, symmetry implies that $\lambda_2(e_1, e_2) = \lambda_1(e_2, e_1)$, $p(\mathbf{e}) = (p_1(e_1, e_2), p_1(e_2, e_1))$, $V^B(e_2, e_1) = V^B(e_1, e_2)$ and $V^{B,INT}(e_2, e_1) = V^{B,INT}(e_1, e_2)$.

[10] The Bellman equation at the price-setting stage is $V_1^S(\mathbf{e}) = \max_{p_1} D_1(p_1, p_2(\mathbf{e}), \mathbf{e})(p_1(\mathbf{e}) - c_1(e_1)) + \sum_{k=0,1,2} D_k(p_1, p_2(\mathbf{e}), \mathbf{e}) V_1^{S,INT}(\mathbf{e}'_k)$, from which equation (1) can be derived by substituting in the prices implied by the first-order conditions. Similarly, equation (2) can be derived from a Bellman equation that determines the continuation choice.

Value for representative buyer before entry/exit stage ($V^{B,INT}$) :

$$V^{B,INT}(\mathbf{e}) - \beta \left(\sum_{\mathbf{e}'} \Pr(\mathbf{e}'|\mathbf{e}, \lambda_1(\mathbf{e}), \lambda_2(\mathbf{e})) V^B(\mathbf{e}') \right) = 0. \tag{6}$$

where $\mathbf{e}'$ are the states the game can evolve to depending on the entry and exit choices of the sellers.

Given the definition of $V^{B,INT}$, the probability that the buyer purchases from seller i is

$$D_i(p, \mathbf{e}) = \frac{\exp(v_i - p_i + V^{B,INT}(\mathbf{e}'_i))}{\sum_{k=0,1,2} \exp(v_k - p_k + V^{B,INT}(\mathbf{e}'_k))}. \tag{7}$$

with a similar formula for the probability of choosing the outside option.

Beginning of period representative buyer value (V^B) :

$$V^B(\mathbf{e}) - b^p \log \left(\sum_{k=0,1,2} \exp\left(v_k - p_k + V^{B,INT}(\mathbf{e}'_k)\right) \right) - (1 - b^p) \sum_{k=0,1,2} D_k(p(\mathbf{e}), \mathbf{e}) V^{B,INT}(\mathbf{e}'_k) = 0, \tag{8}$$

where the last term is the continuation value for a non-chosen buyer, and the second term is the expected value, reflecting both the expected flow utility and the continuation value, for a chosen buyer.

Discussion of b^p. b^p is a buyer's expected share of future buyer surplus, or, equivalently, the proportion of a purchase choice's effect on future buyer surplus that a buyer internalizes. If $b^p = 0$, $V^B = V^{B,INT} = 0$ for all states and the model is equivalent to BDK. If $b^p = 1$, the model is consistent with LY's assumption of a single repeat buyer. If $\frac{1}{b^p}$ is an integer, the model is consistent with a pool of this number of symmetric buyers from which a buyer is chosen with replacement each period (e.g., 5 buyers if $b^p = 0.2$). However, we will vary b^p continuously, and one can rationalize values where $\frac{1}{b^p}$ is not an integer using a behavioral interpretation where all buyers over- or under-estimate their future importance.

b^p only affects purchase probabilities in states where the purchase choice that is made can affect the state that the industry will be in the next period. In particular, the $V^{B,INT}$

terms will cancel in (7) in states $(M,0)$ and (M,M) and a buyer will maximize its current flow utility.

Types of Equilibria. It is useful to distinguish different types of equilibria. Following BDK, equilibria where sellers never exit from duopoly states are "accommodative".

Definition *An equilibrium is* ***accommodative*** *if* $\lambda_1(e_1,e_2)=\lambda_2(e_1,e_2)=1$ *for all states* (e_1,e_2) *where* $e_1>0$ *and* $e_2>0$.

As active sellers always have a positive probability of making a sale, a game that begins with duopoly must end up in absorbing state (M,M) when an equilibrium is accommodative. It is theoretically possible for more than one accommodative equilibrium to exist, although we have never found an example of this type of multiplicity.[11]

Non-accommodative equilibria may take many forms. For policy purposes, one might be particularly interested in equilibria where one seller can become a permanent monopolist (i.e., $(M,0)$ or $(0,M)$ are absorbing states that can be reached with positive probability). We will pay particular attention to a subset of this type of equilibria.

Definition *A symmetric equilibrium has the "****S****ome* ***E****xit* ***L****eads to* ***P****ermanent* ***M****onopoly" (SELPM) property if there is some state* $e_1^*>1$, *where (i)* $\lambda_1(e_1,e_2)=1$ *for all* $e_1\geq e_1^*$[12] *and* $\forall e_2$, *including* $e_2=0$; *(ii)* $\lambda_2(e_1^*,e_2)<1$ *for some* e_2 *where* $0<e_2<e_1^*$, *and* $\lambda_2(e_1,0)=0$ *for all* $e_1\geq e_1^*$.

In words, an equilibrium is SELPM if the leader will not exit for the rest of the game once it has attained a certain level of know-how (e_1^*) (condition (i)), but there is a non-zero probability that a laggard seller 2 will exit in which case there will be no re-entry (condition (ii)). Therefore, once e_1^* has been reached the game will either evolve, with both sellers accumulating know-how, to (M,M), and stay there, or seller 2 may exit in which case the

[11] However, the form of demand implies that accommodative equilibrium prices in state (M,M) must be unique for all b^p.

[12] Note, that as we are only looking at symmetric equilibria, this condition implies that $\lambda_2(e_1,e_2)=1$ for all $e_2\geq e_1^*$.

game will evolve, with seller 1 accumulating know-how, to $(M, 0)$, and stay there.[13] As noted previously, purchase choices do not change the state in these states, so firms will set static Nash prices in these states for all values of b^p.[14]

In Sections 3 and 4 we will note that, for the parameters considered, all of the non-accommodative equilibria identified using homotopies are SELPM. In Section 5.1 we will use a recursive algorithm that can identify if SELPM equilibria exist.

3 The Effects of Strategic Buyers on Equilibrium Outcomes: An Illustration

In this section, we use the parameter values assumed by BDK in their leading example, including $\sigma = 1$ and $\rho = 0.75$, to examine how varying b^p from 0 to 1 changes incentives and equilibrium outcomes.[15] BDK1 argue that these parameters are empirically plausible, although they are not intended to match any particular industry.[16] We will call these the "illustrative parameters". The methods that we use to find equilibria are described in Online Appendix A.

Table 1 shows equilibrium strategies, for a subset of states, for the three equilibria ("baseline equilibria") that both our analysis and BDK identify when $b^p = 0$. One equilibrium is accommodative and the other two are SELPM.[17] The three equilibria differ only in states where at least one firm is in states 0 or 1, with lower SELPM duopoly prices, consistent with the sellers recognizing that a seller that has made no sales may exit.

BDK measure long-run market structure using the long-run HHI (HHI^∞)

$$HHI^\infty = \sum_{\mathbf{e} \neq (0,0)} \frac{\mu^\infty(\mathbf{e})}{1 - \mu^\infty(0,0)} HHI(\mathbf{e}) \text{ where } HHI(\mathbf{e}) = \sum_{i=1,2} \left(\frac{D_i(p, \mathbf{e})}{D_1(p, \mathbf{e}) + D_2(p, \mathbf{e})} \right)^2$$

[13] In a SELPM equilibrium the game must eventually end up in one of these two absorbing states unless $(0, 0)$ is also an absorbing state, in which case the game could end with a completely inactive industry. We have not found an equilibrium where $(0, 0)$ is absorbing for any of the parameterizations considered in this paper.

[14] Note that a monopolist would still set a static price in state $(M, 0)$ even if there was a possibility of re-entry so that $(M, 0)$ was not absorbing.

[15] All of the other parameters take the values noted in Section 2.

[16] Ghemawat (1985) reports the average estimated ρ across 97 empirical studies to be 0.85, with 79 estimates between 0.75 and 0.9.

[17] Any $e_1 = 2, .., 30$ meets the SELPM criteria in both cases.

Table 1: Baseline Equilibrium Strategies for a Subset of States for the Illustrative Parameters when $b^p = 0$.

States		Marg. Costs		High-HHI Eqm. HHI^∞ =0.96, P^∞ =8.28				Mid-HHI Eqm. HHI^∞ =0.58, P^∞ =5.77				Accommodative Eqm. $HHI^\infty = 0.5$, P^∞ =5.24			
e_1	e_2	c_1	c_2	p_1	p_2	λ_1	λ_2	p_1	p_2	λ_1	λ_2	p_1	p_2	λ_1	λ_2
1	1	10	10	-34.78	-34.78	0.9996	0.9996	3.27	3.27	1	1	5.05	5.05	1	1
2	1	8.5	10	0.08	3.63	1	0.7799	3.62	4.65	1	0.9998	5.34	6.29	1	1
3	1	7.73	10	0.56	4.15	1	0.7791	3.44	4.95	1	0.9874	5.45	6.65	1	1
3	2	7.73	8.5	5.61	5.94	1	1	5.61	5.94	1	1	5.61	5.94	1	1
4	1	7.23	10	0.80	4.41	1	0.7787	3.38	5.12	1	0.9767	5.51	6.82	1	1
4	2	7.23	8.5	5.55	6.06	1	1	5.55	6.06	1	1	5.55	6.06	1	1
4	4	7.23	7.23	5.65	5.65	1	1	5.65	5.65	1	1	5.65	5.65	1	1
10	1	5.83	10	1.21	4.86	1	0.7778	3.38	5.46	1	0.9586	5.59	7.12	1	1
10	2	5.83	8.5	5.44	6.28	1	1	5.44	6.28	1	1	5.44	6.28	1	1
10	10	5.83	5.83	5.32	5.32	1	1	5.32	5.32	1	1	5.32	5.32	1	1
29	1	3.25	10	1.24	4.90	1	0.7777	3.39	5.49	1	0.9577	5.58	7.15	1	1
29	2	3.25	8.5	5.42	6.30	1	1	5.42	6.30	1	1	5.42	6.30	1	1
30	1	3.25	10	1.24	4.90	1	0.7777	3.39	5.49	1	0.9577	5.58	7.15	1	1
30	2	3.25	8.5	5.42	6.30	1	1	5.42	6.30	1	1	5.42	6.30	1	1
30	29	3.25	3.25	5.24	5.24	1	1	5.24	5.24	1	1	5.24	5.24	1	1
30	30	3.25	3.25	5.24	5.24	1	1	5.24	5.24	1	1	5.24	5.24	1	1
1	0	10	-	8.80	-	1	0	7.55	-	1	0.1357	8.19	-	1	0.8816
2	0	8.5	-	8.72	-	1	0	8.72	-	1	0	8.45	-	1	0.5233
10	0	5.83	-	8.56	-	1	0	8.56	-	1	0	8.55	-	1	0.2953
0	0	-	-	-	-	0.9583	0.9583	-	-	0.9698	0.9698	-	-	0.9552	0.9552
0	1	-	10	-	8.80	0	1	-	7.55	0.1357	1	-	8.19	0.8816	1
0	2	-	8.5	-	8.72	0	1	-	8.72	0	1	-	8.45	0.5233	1
0	3	-	7.73	-	8.68	0	1	-	8.68	0	1	-	8.52	0.4227	1
0	4	-	7.23	-	8.65	0	1	-	8.65	0	1	-	8.54	0.3739	1
0	10	-	5.83	-	8.56	0	1	-	8.56	0	1	-	8.55	0.2953	1
0	29	-	3.25	-	8.54	0	1	-	8.54	0	1	-	8.54	0.2899	1
0	30	-	3.25	-	8.54	0	1	-	8.54	0	1	-	8.54	0.2899	1

Notes: p_i and λ_i are the equilibrium price and equilibrium probability of continuing to be in the industry in the next period of seller i. The calculations of HHI^∞ and P^∞ are described in the text.

where $\mu^\infty(\mathbf{e})$ is the probability that a game beginning in state (1,1) will be in state $\mathbf{e}$ after 1,000 periods, approximating the long-run, given equilibrium strategies. The long-run expected price (P^∞) is defined similarly with the sale probabilities weighting the prices of the active sellers. In an accommodative equilibrium $\mu^\infty(M,M)$ is essentially one, so $HHI^\infty = 0.5$ and $P^\infty = 5.24$. In either of the SELPM equiibria, the game may alternatively end up in absorbing states $(M,0)$ or $(0,M)$, where the $HHI(\mathbf{e})$ is 1 and prices are 8.54, so that HHI^∞ and P^∞ reflect the probabilities of permanent duopoly and permanent monopoly outcomes.

Increasing b^p. We now consider the effect of increasing b^p. We first consider buyer behavior, and how changes in buyer behavior affect seller incentives, and then equilibrium outcomes.

As $\lambda_2(e_1, e_2 > 1) = 1$ in the SELPM equilibria, a buyer in a state $(e_1 > 1, 1)$ can guarantee long-run duopoly if it buys from seller 2 (the laggard). The large difference (79.58) in the present value of buyer surpluses in the absorbing duopoly and monopoly states, (M,M) and $(M,0)$, implies that the incentive for even a moderately strategic buyer to buy from the laggard may be substantial.[18]

Figure 2(a) shows seller 2's demand in state (3,1), holding seller strategies fixed at their baseline equilibrium values, for different values of b^p.[19] As b^p rises, seller 2's demand increases significantly in the SELPM Mid-HHI and High-HHI equilibria even for low values of b^p.[20] For example, at the High-HHI baseline equilibrium price (4.15), the probability that seller 2 wins the sale increases from 0.027 to 0.306 as b^p increases from 0 to 0.1 (seller 1's probability falls from 0.973 to 0.694).

This shift in buyer demand increases seller 2's value. Figure 2(b) shows $V_2^S(3,1)$ holding

[18] Recall that the value of b^p does not affect equilibrium prices in absorbing states. The present value of buyer surplus in state (M,M) is 114.57 ($\frac{\ln(2*\exp(10-5.242)+\exp(0))}{1-\frac{1}{1.05}}$) and the present value in state $(M,0)$ is 34.99 ($\frac{\ln(\exp(10-8.543)+\exp(0))}{1-\frac{1}{1.05}}$). Of course, the fact that prices in states like $(4,1)$ are lower than in states such as $(4,2)$ in the SELPM equilibria partially offsets this incentive.

[19] The figure is drawn varying $p_2(3,1)$ only in the current period i.e., the buyer assumes that p_2 will have its baseline equilibrium value if the game is in state (3,1) in any future period. p_1, λ_1 and λ_2 are held fixed at their baseline equilibrium values in all states.

[20] There is also a small shift in demand towards the laggard in the accommodative equilibrium as accommodative prices are lower when states are more symmetric.

seller strategies fixed at their baseline equilibrium values but allowing demand to change. If $\beta V_2^S(3,1)$ is greater than $(\overline{X}+\Delta_X)$ (maximum scrap value) then seller 2 will never choose to exit in state (3,1). The buyer-demand adjusted $V_2^S(3,1)$s for the High-HHI and Mid-HHI equilibria cross this threshold when $b^p \approx 0.15$ and 0.01 respectively.

The change in demand also affects the sellers' pricing incentives. BDK define two dynamic incentives for a seller.

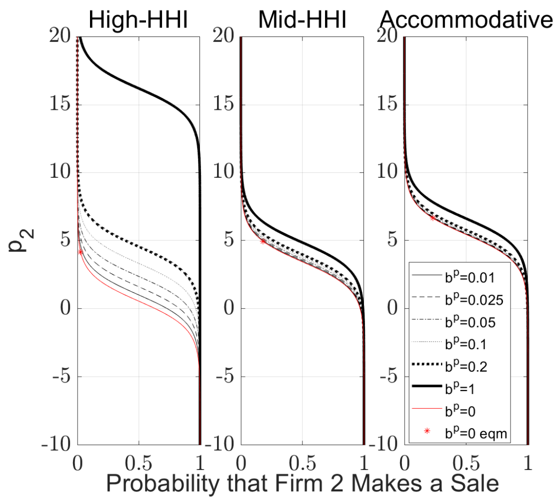
Definition *Seller 1's* ***advantage-building*** *(AB) incentive is* $V_1^{S,INT}(e_1{+}1,e_2){-}V_1^{S,INT}(e_1,e_2)$, *and its* ***advantage-denying*** *(AD) incentive is* $V_1^{S,INT}(e_1,e_2)-V_1^{S,INT}(e_1,e_2+1)$.

BDK identify the advantage-denying incentive as particularly important in sustaining equilibria that can result in monopoly. Figure 2(c) shows how seller 1's incentives change in state $(3,1)$ as b^p increases, holding seller strategies fixed so that changes reflect only changes in demand. Even though seller 2 is still likely to exit if it does not make the sale, seller 1's High-HHI equilibrium AD incentive, which is large when $b^p=0$, falls rapidly as b^p increases, reflecting how seller 2 is more likely to make a sale in future periods if it remains. The other incentives decline only slightly, and more linearly, as b^p increases.

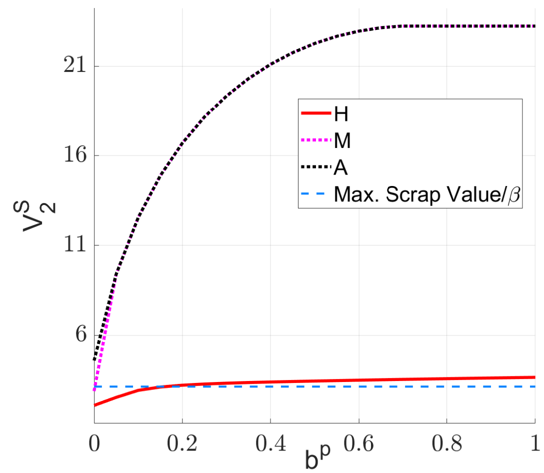
Figure 2(d)-(f) shows state (3,1) prices, seller 2's continuation probability in state (3,1) and the HHI^∞s implied by equilibria when we follow the equilibrium correspondence using b^p-homotopies from each baseline equilibrium (see Appendix D.1 for what happens to incentives). The High (H)- and Mid (M)-HHI baseline equilibria lie at the two ends of a loop (i.e., the homotopies trace the same path in opposite directions) in the equilibrium correspondence that does not extend beyond $b^p=0.142$ (approximately 7 symmetric sellers). All of the equilibria on this loop are SELPM. The homotopy path from the accommodative equilibrium extends to $b^p=1$, and all equilibria on this path are accommodative (i.e., $\lambda=1$ for all duopoly states, and $HHI^\infty=0.5$). We only ever find one accommodative equilibrium so that we have a unique equilibrium for $b^p>0.142$. We will provide additional evidence that there are no SELPM equilibria for $b^p>0.142$ in Section 5.1. The decline in seller 1's demand causes equilibrium prices to initially fall as b^p increases from zero from the H and M equilibria, but sellers' prices rise on the path from the accommodative equilibrium as both sellers' incentives to gain an advantage are weakened by how strategic buyers tend to favor

Figure 2: Demand, Dynamic Incentives, Equilibrium Strategies and Long-Run Market Concentration for the Illustrative Parameters. H = High-HHI, M = Mid-HHI and A = Accommodative Baseline Equilibria, and AB=Advantage-Building and AD=Advantage-Denying Incentives. Panels (a)-(c) hold seller strategies fixed at baseline equilibrium values, and panels (d)-(f) show equilibrium strategies and implied HHI^{∞} along b^p-homotopy paths from the A equilibrium (black lines) and overlapping paths from the H and M equilibria (red lines).

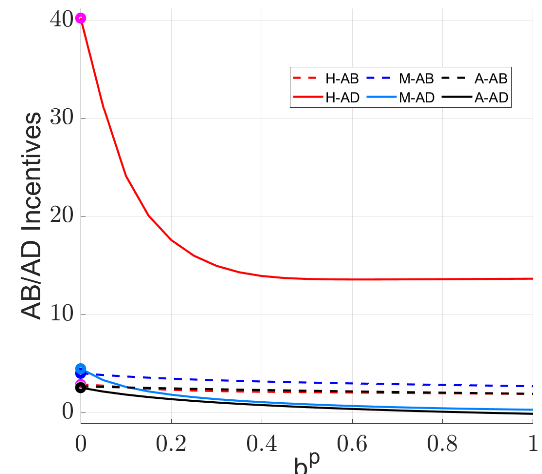
(a) Inverse Demand for Seller 2 in (3,1).



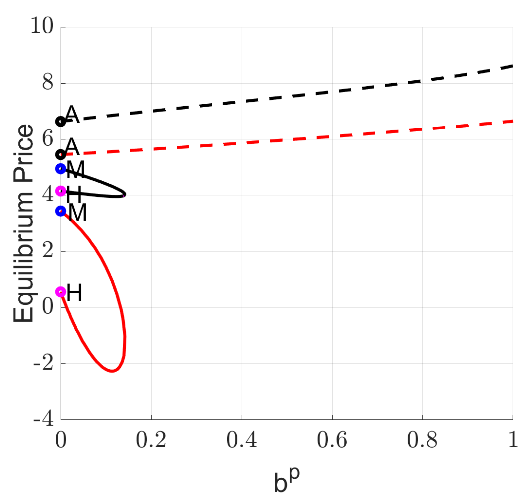
(b) Seller 2 Value in State (3,1).



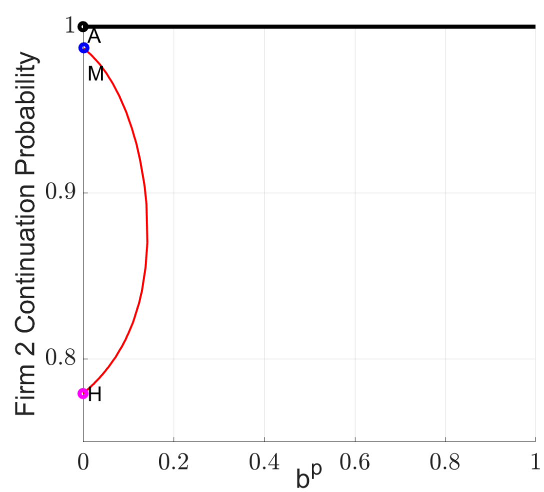
(c) Seller 1 Pricing Incentives.



(d) State (3,1) Equilibrium Prices (red (black)=firm 1 (2)).



(e) State (3,1) Seller 2 Equilibrium Continuation Probability.



(f) Expected Equilibrium Long-Run HHI (HHI^{∞}).

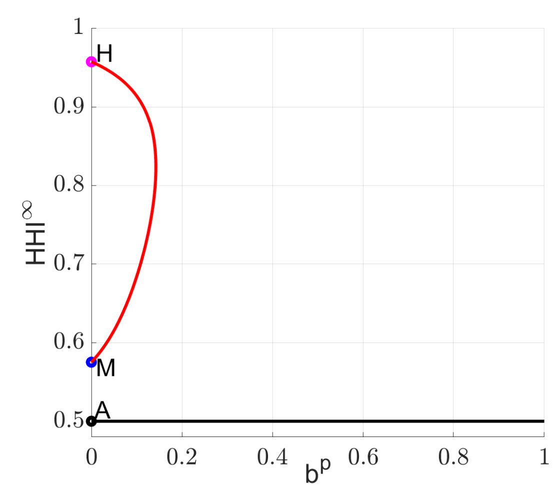
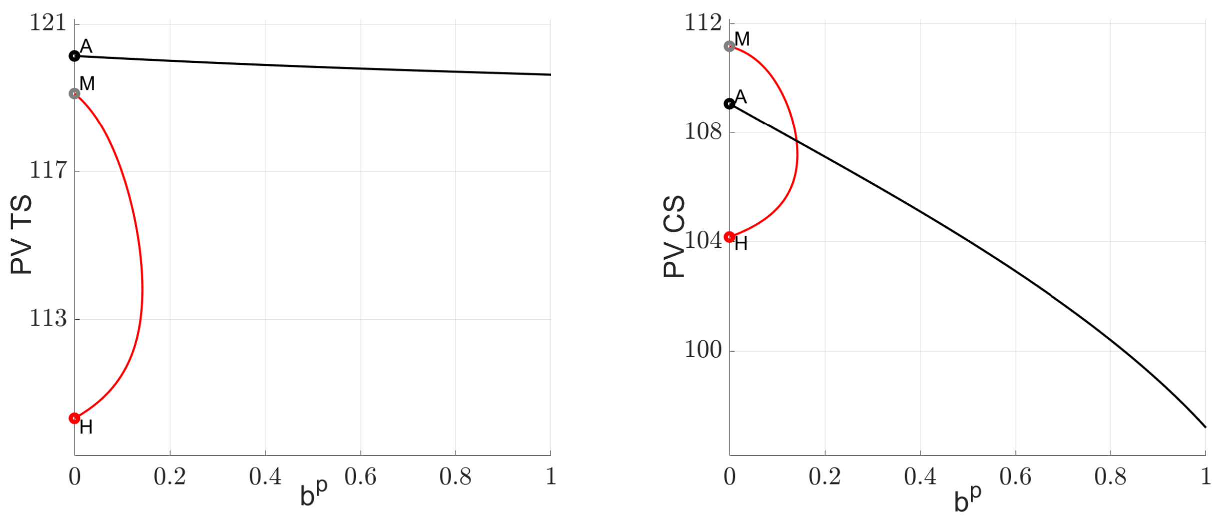


Figure 3: Present Value of Surplus for the Illustrative Parameters along b^p-Homotopy Paths. H = High-HHI, M = Mid-HHI and A = Accommodative Baseline Equilibria. The black line traces the homotopy path from the Accommodative (A) baseline equilibrium. The red line traces the overlapping paths from the High-HHI (H) and Mid-HHI (M) baseline equilibria.

(a) PV Total Surplus. (b) PV Consumer/Buyer Surplus.



the laggard.

Figure 3 shows what happens to the present value of equilibrium expected total surplus (PV TS) and buyer surplus (PV CS) for a game starting in state (1,1). The long-run values of both measures are higher in the accommodative equilibrium, but lower initial prices can raise present values in the SELPM equilibria. The accommodative equilibria have higher PV TS, but, when multiple equilibria exist, the PV CS of the accommodative equilibrium lies between the PV CSs of the SELPM equilibria.[21] Therefore, strategic buyer behavior can actually eliminate a type of equilibrium that produces more surplus for buyers, and, within the type of equilibrium that survives, increasing b^p can lower buyer welfare.[22]

[21] Appendix D.2 further explores the welfare patterns by examining what happens to the number of sales and production costs, in the long-run and in the first ten periods of the game. Even though one seller is likely to exit, expected surplus is highest in the High-HHI equilibrium in the first ten periods because duopoly prices are so low.

[22] These results are potentially relevant for whether buyers who do not compete downstream might have an incentive to merge before the game starts in order to prevent the upstream industry possibly ending up in monopoly. Even ignoring possible costs of agreeing to a merger, there would be no incentive for all buyers to merge in this example if, absent a merger, the Mid-HHI equilibrium would be played. If the High-HHI equilibrium would be played, there would not be an incentive to merge to monopsony, as this would lower PV CS, and it might not be attractive for a subset of buyers to merge, without a subsidy from the remaining buyers who might capture many of the benefits from the merger.

4 Results Across Values of ρ and σ

In this section, we examine whether strategic buyer behavior changes equilibrium outcomes in similar ways for different values of ρ, the progress ratio and σ, the degree of product differentiation. Lower σ reduces long-run duopoly prices and profits, which tends to lead to more exit, while slightly increasing monopoly profits, which may increase the competition to become a monopolist. Lower ρ tends to raise duopoly profits but it also gives the seller that makes the first sale a larger cost advantage, so that the effect on the incentives of a laggard to exit are ambiguous. We find equilibria by running σ- and ρ-homotopies for different, discrete, values of b^p, holding all of the other parameters fixed at their illustrative values.[23]

Figure 4(a) and (b) show the values of HHI^∞ and P^∞ implied by the equilibria on σ-homotopy paths ($\rho = 0.75$) for 11 different values of b^p.[24] All of the equilibria identified are accommodative or SELPM, and we only ever find one accommodative equilibrium for given parameters. For $b^p = 0$ (black solid line, shown on its own in Appendix D.3 for clarity), we find a single, accommodative equilibrium when $\sigma > 1.12$ (high differentiation), but for lower σ, we find that at least one equilibrium exists where a duopolist may exit and for some values there are many equilibria. For example, there are 23 equilibria for $\sigma = 0.8$, all of which have $HHI^\infty \geq 0.95$ and very similar $P^\infty > 8.5$. When accommodative and SELPM equilibria coexist, the accommodative equilibrium has the lowest P^∞.

The σ-homotopy paths unwind as b^p rises, which tends to reduce multiplicity but also leads to accommodative equilibria existing for lower σ. The probability of monopoly, reflected in HHI^∞, in the SELPM equilibria tends to fall. While multiplicity was eliminated when $b^p > 0.142$ for $\sigma = 1$, we find a similar result, but with a higher b^p threshold, when there is less differentiation. For example, we find a unique (accommodative) equilibrium for $\sigma = 0.8$ only when $b^p \geq 0.5$.

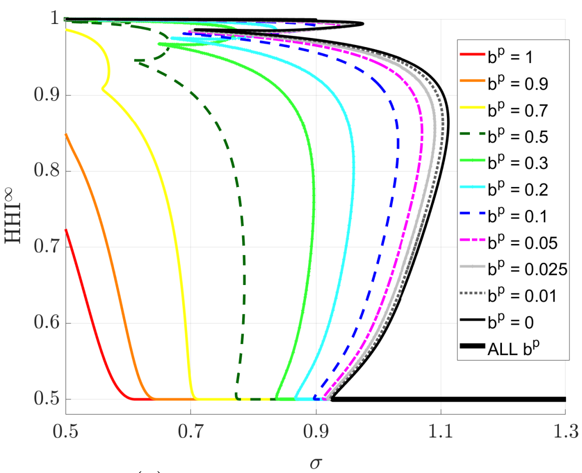
Figures 4(c) and (d) show similar plots for ρ-homotopy paths, with the other parameters at their illustrative values, including $\sigma = 1$. The x-axis is ordered so that LBD increases

[23] Varying other parameters may also generate interesting effects. However, we think it is natural to focus on the progress ratio, which measures the extent of LBD, and product differentiation, which distinguishes the CR and BDK models from earlier analyses of LBD and market structure, such as Dasgupta and Stiglitz (1988).

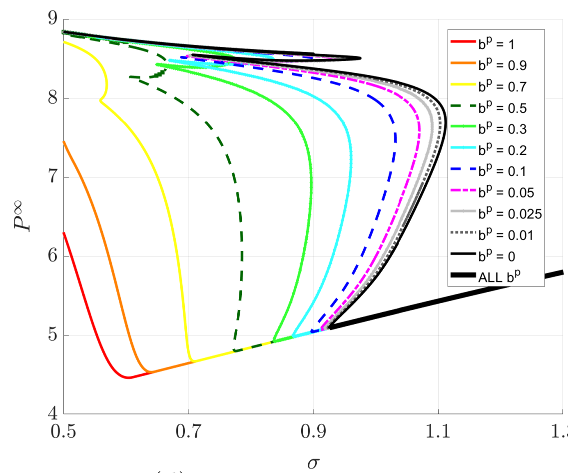
[24] For all b^p, we begin the path at $\sigma = 1.3$ where we find what appears to be a unique equilibrium by solving the equilibrium equations.

Figure 4: Equilibrium HHI^{∞}, P^{∞} and Present Value of Surplus on σ or ρ-Homotopy Paths for Different b^p, with Other Parameters at their Illustrative Values. Surplus in panels (e) and (f) is measured relative to surplus in the accommodative equilibrium when $b^p = 1$, and solid (dashed) lines indicate accommodative (non-accommodative) equilibria. The "All b^p" line indicates accommodative equilibria for b^p values 0, 0.01, 0.025, 0.05, 0.1, 0.2, 0.3, 0.5, 0.7, 0.9 and 1.

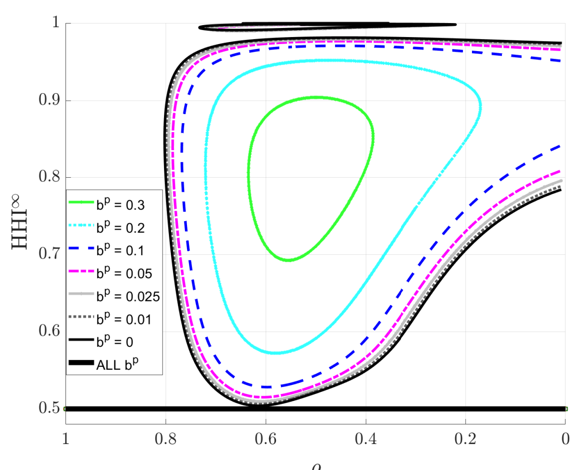
(a) σ-Homotopies: HHI^{∞}.



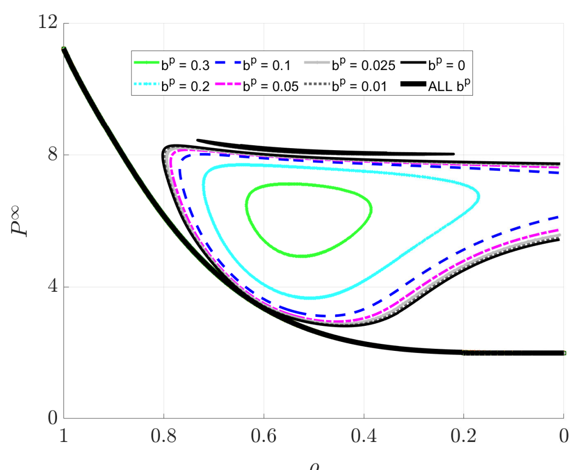
(b) σ-Homotopies: P^{∞}.



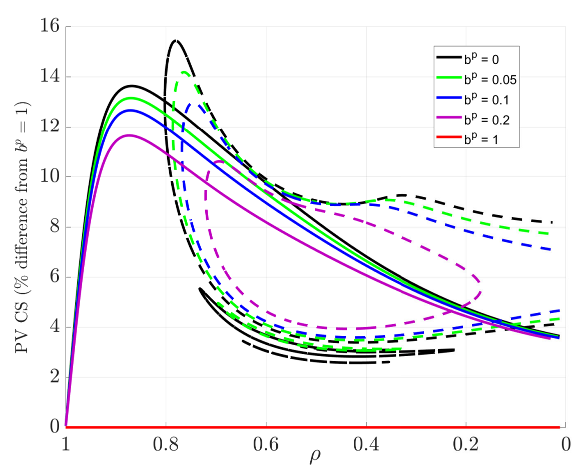
(c) ρ-Homotopies: HHI^{∞}.



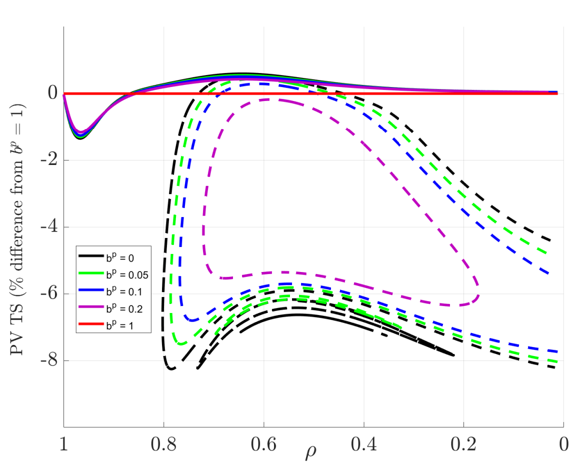
(d) ρ-Homotopies: P^{∞}.



(e) ρ-Homotopies: PV CS.



(f) ρ-Homotopies: PV TS.



to the right. We find one accommodative equilibrium for all ρ and for all b^p. All identified non-accommodative equilibria are SELPM. For $\rho > 0.803$, which is very relevant empirically (see footnote 16), we find only an accommodative equilibrium for all b^p. For lower ρ, we find that SELPM and accommodative equilibria co-exist when b^p is small enough. For $b^p = 0$, the SELPM equilibrium correspondence has (in this dimension) two disconnected loops. The loop with the highest HHI^∞/P^∞s is eliminated for $b^p \geq 0.05$, and the second loop contracts as b^p rises, disappearing entirely for $b^p > 0.3$, so that only accommodative equilibria remain.

Figures 4(e) and (f) show PV TS and PV CS for the ρ-homotopies (Appendix D.4 shows the figures for the σ-homotopies). The patterns are broadly consistent with the illustrative parameter example. When accommodative and SELPM equilibria coexist, the accommodative equilibrium has higher PV TS, while its PV CS lies between the values of the SELPM equilibria. BDK2 argue that equilibria are quite efficient in the BDK model, and this conclusion tends to be strengthened when buyers are strategic in the sense that less efficient types of equilibria are eliminated. Increasing b^p lowers PV CS in accommodative equilibria, as initial price competition is softened, and it tends to lower PV TS for $\rho \leq 0.9$. For high ρ the pattern is different, as costs are sufficiently high that the probability that non-strategic buyers make no purchase is not negligible, which inefficiently slows the industry's progress down, but this probability tends to fall when buyers are strategic.[25]

5 Robustness Checks, Extensions and Discussion

The results presented so far suggest that, for empirically relevant progress ratios, moderately strategic buyer behavior eliminates the multiplicity of equilibria that is common when buyers are atomistic and, in particular, tends to eliminate equilibria that are likely to result in long-run industry domination by a single seller. However, the limitations of the method used to find equilibria and the simplicity of the model may provide reasons for caution. In this section, we explore and discuss the robustness of our results.

[25]For example, when $\sigma = 1$, $\rho = 0.925$ and $b^p = 0.2$, the probability that the buyer chooses the outside option in state $(1, 1)$ is around 0.272. When $b^p = 1$, this probability falls to 0.241 (a 10% decrease).

5.1 Alternative Method for Identifying SELPM Equilibria

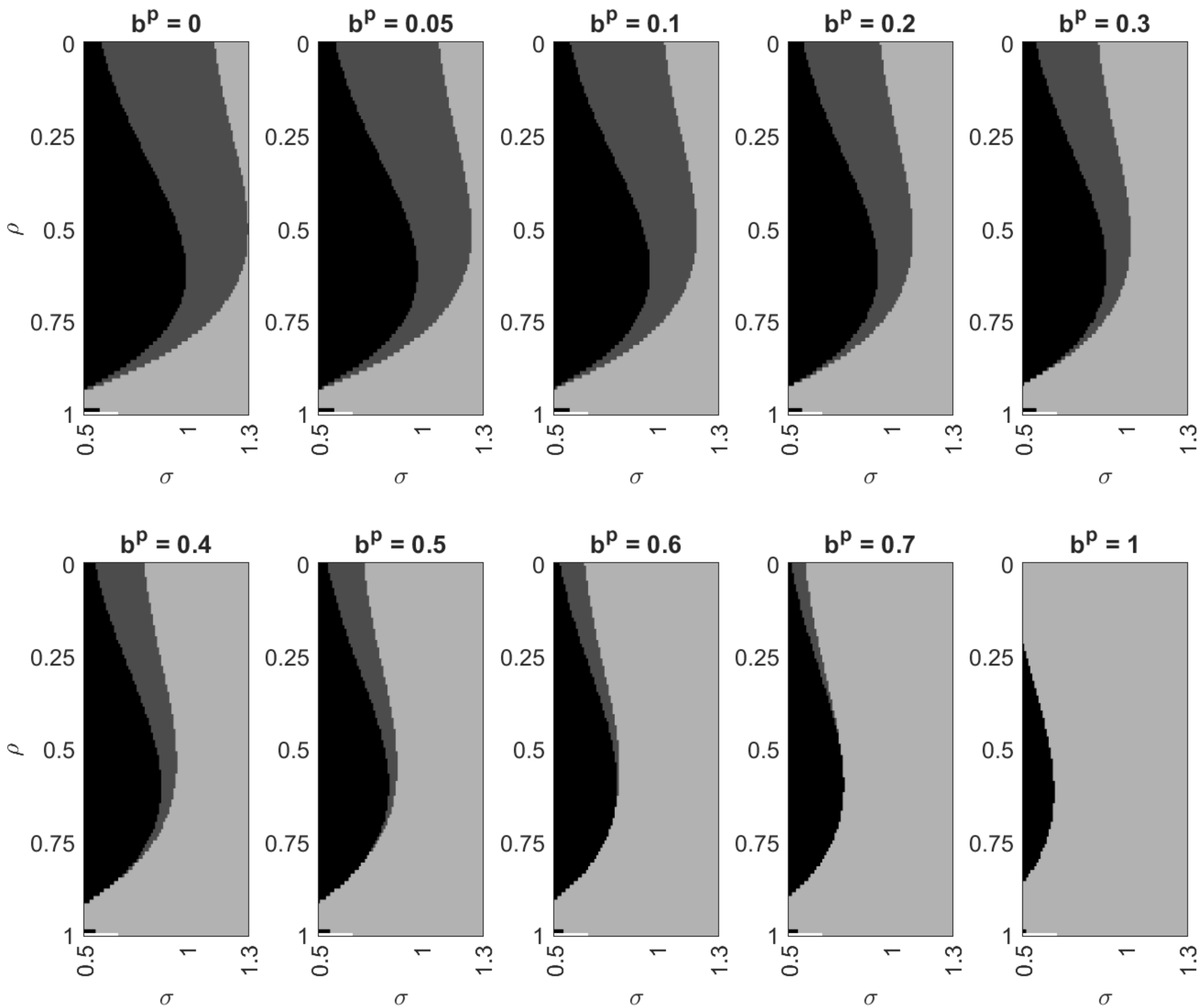
Homotopies are only guaranteed to be able to find all equilibria under particular restrictions that our model does not satisfy (Judd, Renner, and Schmedders (2012)), so our results could potentially reflect a systematic failure to find non-accommodative equilibria when buyers are strategic. As a check, we therefore use an alternative algorithm that can identify whether SELPM equilibria exist for given parameters, exploiting the feature that, in a SELPM equilibrium, once a state e_1^* has been reached, the state will transition to either (M, M) or $(M, 0)$ without returning to a previously visited state. This feature implies that an algorithm that works backwards from (M, M) and $(M, 0)$ will be able to find an e_1^* state, if one exists, as long as we can find all SELPM-consistent equilibria in a given state given continuation values if a state changes. Appendix B describes the algorithm and the conditions under which it will work.[26] Appendix C describes a simpler algorithm that can identify if an accommodative equilibrium exists.

To be clear, the algorithm cannot determine if non-SELPM non-accommodative equilibria exist. However, as all of the non-accommodative equilibria identified by the Section 3 and 4 homotopies are SELPM, proving that no SELPM equilibra exist for given parameters provides, at least, highly suggesting evidence that an accommodative equilibrium, if one exists, is likely unique.

Figure 5 shows the types of equilibria that we find exist for a grid of values of (b^p, ρ, σ) with the other parameters at their illustrative values. We highlight three results. First, for $\rho = 0.75$ or $\sigma = 1$, the results are completely consistent with those presented in Sections 3 and 4 suggesting that homotopies are an effective way to find SELPM and accommodative equilibria for all b^p. Second, there is a small set of parameters with no LBD and low differentiation ($\rho = 1, \sigma \leq 0.65$) where equilibria must be non-accommodative and non-SELPM. For these parameters, the present value of perpetual duopoly profits in state (M, M) is less than the highest possible scrap value, so (M, M) cannot be an absorbing state. Third, for the remaining combinations, the qualitative pattern is consistent with our earlier findings.

[26] Note that the algorithm does not identify how many SELPM equilibria there may be, partly because the recursive algorithm cannot find equilibrium strategies in parts of the state space where there can be exit followed by re-entry.

Figure 5: Classification of the Types of Equilibria that Exist for (b^p, ρ, σ) Combinations with Remaining Parameters at their Illustrative Values. ρ and σ are varied in steps of 0.01. The algorithm for identifying if SELPM (accommodative) equilibria exist is described in online Appendix B (C). Shading: White - neither SELPM nor accommodative equilibria exist; Light Grey - an accommodative equilibrium exists, no SELPM equilibria exist; Dark Grey - an accommodative equilibrium and SELPM equilibria co-exist; Black - SELPM equilibria exist, no accommodative equilibrium exists.



Accommodative and SELPM equilibria co-exist over a wide range of the parameter space when $b^p = 0$. Accommodative (SELPM) equilibria are supported for wider (narrower) ranges of parameters as b^p rises. However, for $0.2 < \rho < 0.9$, SELPM equilibria can exist even when $b^p = 1$ when there is minimal product differentiation.

5.2 Mixture of Strategic and Non-Strategic Buyers.

The Section 2 model assumes that all buyers are equally strategic, whereas it may be more common that there are some repeat purchasers and some buyers that expect to be in the market only once. To investigate how our results may change if strategic and non-strategic buyers coexist, we solve, for the illustrative parameters, an extended version of our model with four symmetric strategic buyers.[27] Nature chooses a non-strategic buyer each period with probability $(1-\gamma)$, and otherwise randomly chooses one of the strategic buyers. Sellers can set different prices depending on the buyer's type. If $\gamma = 0$ then the model is equivalent to the original BDK model, with prices equal to those in the baseline equilibria. The details of this extension, and the next three extensions, are provided in Online Appendix E.

Figure 6(a) shows the HHI^{∞} implied by the equilibria on γ-homotopy paths that start from the $\gamma = 0$ equilibria. We find accommodative equilibria for all γ and non-accommodative equilibria, all of which are SELPM, when $\gamma \leq 0.79$. If $\gamma = 0.79$, each strategic buyer expects to be the buyer with probability $\frac{\gamma}{4} = 0.198$ in future periods. This is greater, but not too much greater, than the threshold probability of 0.142 which eliminated SELPM equilibria in the Section 2 model, suggesting that the existence of non-strategic buyers may require strategic buyers to behave "more strategically" to eliminate SELPM equilibria.

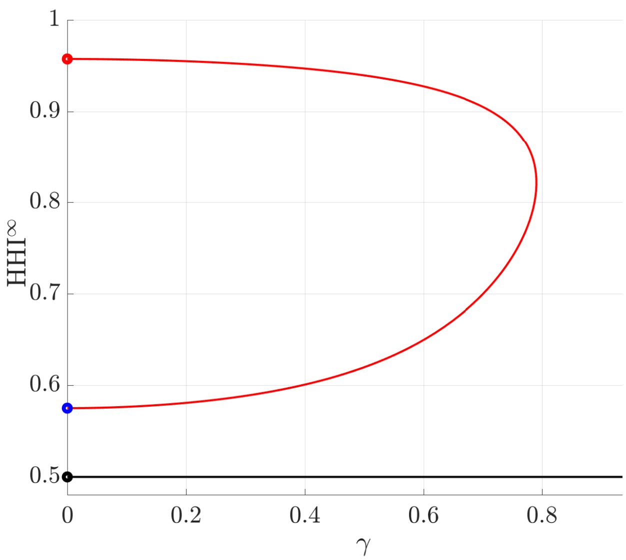
5.3 Buyers with Persistent Preferences Over Sellers.

The Section 2 model also assumes that all buyers have identical preferences over sellers up to iid preference shocks. In reality, some buyers may have systematic preferences for a particular seller (for example, because of geographic location or compatibility with existing equipment). We therefore extend the Section 2 model by assuming that there are equal

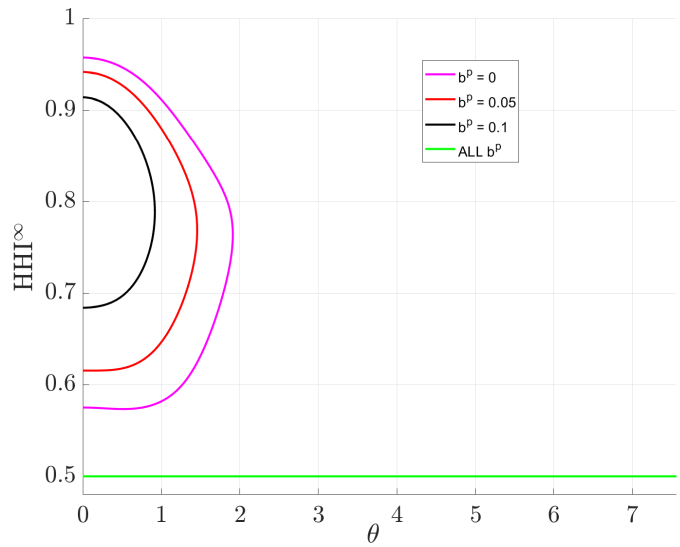
[27] Patterns are qualitatively similar for different numbers of strategic buyers.

Figure 6: Equilibrium Expected Long-Run HHI (HHI^{∞}) for Various Extensions.

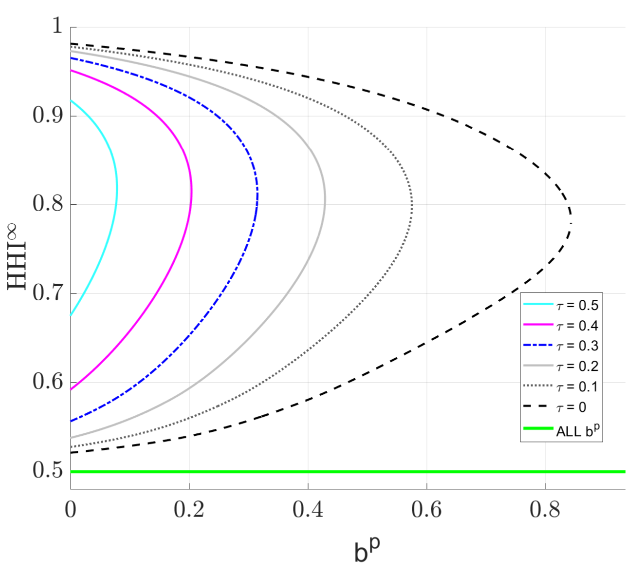
(a) Mixture of Strategic and Non-Strategic Buyers: γ-Homotopies. Red line indicates paths from non-accommodative equilibria. Black line indicates path from accommodative equilibrium.



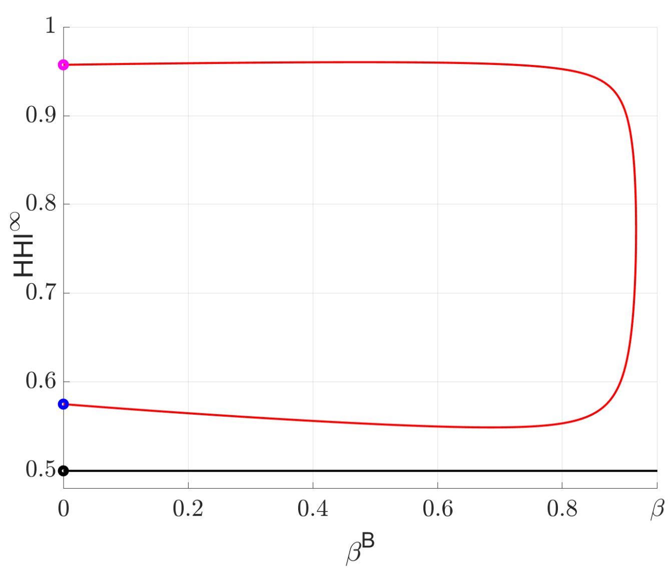
(b) Buyers with Different Seller Preferences: θ-Homotopies for $b^p = 0, 0.05$ and 0.1. "ALL b^p" includes these values and values of b^p above 0.15.



(c) Bargaining as a Constraint on Monopoly Power: b^p-Homotopies. "ALL b^p" refers to values 0, 0.1, 0.2,..., 0.9 and 1. τ is the Nash bargaining parameter that indicates the buyer's share of surplus.



(d) Variable Buyer Discount Factors: β^B-Homotopies for $b^p = 1$. Red line indicates paths from non-accommodative equilibria. Black line indicates path from accommodative equilibrium.



numbers of two types of buyers. Type 1's indirect utility when it purchases from sellers 1 and 2 respectively are $v_1 + \frac{\theta}{2} - p_1 + \epsilon_1$ and $v_2 - \frac{\theta}{2} - p_2 + \epsilon_2$. For type 2 buyers, the signs on the $\frac{\theta}{2}$ terms are reversed. Sellers recognize the type of the buyer before setting prices. The model is equivalent to Section 2 model when $\theta = 0$. Intuitively, as θ increases it will become more attractive for a seller that has a marginal cost disadvantage to remain in the market as it will still have an advantage when selling to half of the market.

Figure 6(b) shows, for the illustrative parameters, the HHI^{∞} implied by equilibria on θ-homotopy paths that start at the $\theta = 0$ equilibria for $b^p = 0, 0.05$ and 0.1, values that support multiple equilibria when $\theta = 0$.[28] There are accommodative equilibria for all θ, but the non-accommodative equilibria, all of which are SELPM, are eliminated for relatively low θs, especially when buyers are strategic.[29] Therefore, less strategic behavior may be required to generate our qualitative results than in our simple model when buyers have persistent preferences.

5.4 Bargaining as a Constraint on Monopoly Power.

When a low know-how laggard may exit, a strategic buyer has an incentive to buy from the laggard in order to reduce the probability that it will be exposed to monopoly power in future periods. However, at least two considerations might make a large buyer less concerned about a monopoly outcome. First, in the spirit of Aghion and Bolton (1987), Rasmusen, Ramseyer, and Wiley (1991) and Segal and Whinston (2000), the leader might sign multi-period contracts with large buyers, which simultaneously protect these buyers from future monopoly power while also making it less profitable for the laggard to remain in the market. We view the relaxation of the period-by-period price competition assumption of BDK, CR and LY to allow for contracts as an important next step in this research, although contracts may provide imperfect protection from a monopolist when products are complicated and/or customized.

Second, even if we assume period-by-period competition, it is possible that a buyer would

[28] The interpretation of b^p is still the unconditional probability with which a given buyer expects to be the chosen buyer in any future period, so that a model with a single buyer of each type would have $b^p = 0.5$.

[29] To put the magnitude of θ in context, a seller's marginal cost drops by 1.5 with its first sale for these parameters.

be able to negotiate with a monopoly seller rather than being faced with a take-it-or-leave-it price. If negotiations partially protect buyers this could make them less concerned with preserving competition, but it could also provide a leader with less incentive to try to become a monopolist. To provide a preliminary assessment of how these forces play out, we adjust the Section 2 model by assuming that, in monopoly states, a strategic buyer and a monopolist play a complete information Nash bargaining game (i.e., the buyer ϵ preferences become observed) where, in each period, the buyer receives a share τ of the surplus from trade. The change in the assumed information structure means that BDK's model is no longer a special case even when $b^p = 0$. However, as a comparison, the expected transaction price in state (30,0) is approximately the same as in the baseline equilibria when τ is slightly greater than 0.2.

Figure 6(c) shows, for the illustrative parameters, the HHI^{∞} implied by equilibria on b^p-homotopy paths for different values of τ. An accommodative equilibrium exists for all considered (τ, b^p) combinations, and we find only accommodative equilibria when $\tau \geq 0.6$. When a monopolist seller and a buyer have equal bargaining power (i.e., $\tau = 0.5$), we find only accommodative equilibria when $b^p \geq 0.08$, which is a lower threshold than we identified for our basic model.

5.5 Buyer Discount Factors.

It may be tempting to believe that b^p can also be interpreted as buyer patience, as the $b^p = 0$ equilibria would still be equilibria if there was a myopic monopsonist.[30] However, increasing a buyer discount factor (call this β^B) from zero has a different effect to increasing b^p because, for low β^B, the buyer will care primarily about surplus in the immediate future, and, in non-accommodative equilibria, this is often increased by buying from the leader. This is illustrated in Online Appendix Figure E.1 which shows that, for baseline equilibrium seller strategies, increasing β^B when $b^p = 1$ tends to move demand away from seller 2 in state (3,1) in the Mid- and High-HHI equilibria, until $\beta^B \geq 0.5$, reflecting how prices are significantly lower in state (4,1) than state (3,2). This is the opposite of the pattern when

[30]We thank David Besanko for asking a question that led us to realize that one should not interpret b^p in terms of buyer patience.

b^p increases from zero (Figure 2(a)).

Figure 6(d) shows the HHI^∞ implied by equilibria on the β^B-homotopy paths from the baseline equilibria when we assume $b^p = 1$, limit $\beta^B \leq \beta = \frac{1}{1.05}$ and other parameters have their illustrative values. Accommodative and non-accommodative equilibria coexist until β^B is almost equal to β, which is a qualitatively different pattern to the elimination of these equilibria for low b^p in our model. While there may be industries where buyers are less patient than sellers, it seems plausible that buyers and sellers have similar time preferences in most industries where LBD has been identified, even if each buyer knows it will only account for a proportion of future demand.

One might also wonder what happens if we were to change the seller discount factor. Holding the scrap value distribution fixed, seller values and the attractiveness of remaining in the market increase as β tends towards 1, which will also eliminate non-accommodative equilibria even if $b^p = 0$.

5.6 Forgetting.

Our model follows CR and BDK in assuming that sellers can only lose know-how by exiting the industry. However, Benkard (2000) and Thompson (2007) provide empirical evidence that know-how can also depreciate when production slows ("forgetting"). Besanko, Doraszelski, Kryukov, and Satterthwaite (2010) (BDKS) show that a model where duopolists can stochastically forget but cannot exit also has multiple equilibria that result in different expected levels of long-run industry concentration. One might expect strategic buyer behavior to have less effect on equilibria in the BDKS model because depreciation may eliminate the know-how that a laggard gains through a sale. However, our working paper, Sweeting, Jia, Hui, and Yao (2021) shows that for many values of ρ and alternative forgetting probabilities, multiplicity of equilibria and equilibria that tend to lead to the most asymmetric long-run market structures are eliminated for lower values of b^p than in the BDK model.[31]

[31] However, there are also small groups of parameters where increases in b^p increase the number of equilibria.

6 Conclusion

We have provided a tractable framework for analyzing how equilibrium strategies and market outcomes change when buyers partially internalize how their purchase decisions affect future surplus, in the context of a well-known dynamic model where sellers benefit from LBD. Our framework allows for an investigation of what happens between the polar cases of short-lived atomistic buyers and monopsony, motivated by the fact that many industries where cost-side dynamics are important have at least some large and repeat customers. Our main finding is that, for many empirically relevant parameters, even moderately strategic buyer behavior can eliminate equilibria where the market may come to be dominated by a single firm.

We view this result as having implications for anti-predation policies that have to strike a delicate balance between the potentially large benefits of preserving competition and the risk that intervention will deter pro-competitive pricing. Our results suggest that the existence of equilibria where an industry may become a monopoly will depend on the incentives of customers to offset predatory behavior, and it may be appropriate to treat claims of predation more skeptically when there are several large, repeat customers.

We believe that our framework can be usefully applied to investigate the effects of strategic behavior in settings where incumbents' advantages may arise from other sources, such as network effects or switching costs, or where dynamics arise from the durable or perishable nature of products. While we have investigated some alternative specifications, we view understanding how the ability of sellers to offer multi-period contracts to some customers would affect our results as an important next step of this research. We also believe that, in some settings, it may be useful to include strategic buyers in empirical models of dynamic competition, as doing so may not only make these models more realistic but also help to reduce concerns that multiple equilibria may make it hard to interpret counterfactuals.

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ONLINE APPENDICES FOR "DYNAMIC PRICE COMPETITION, LEARNING-BY-DOING AND STRATEGIC BUYERS" BY SWEETING, JIA, HUI AND YAO

A Methods for Finding Equilibria

In this Appendix we describe the two methods that we use to find equilibria for our analyses in Sections 3, 4 and 5 (excepting Section 5.1).

A.1 Equation Solving.

One method for finding an equilibrium for a fixed set of parameters is to numerically solve the 4,743 value, continuation probability and first-order condition equations in Section 2 using the `fsolve` tool in MATLAB.[32] We specify tolerances of 1e-14 on the variables and on the objective function.

We use equation solving to identify equilibria from which we can start homotopies, and also, as we describe below, to fill in any gaps in a homotopy path that results from the homotopy algorithm stalling.

A.2 Homotopies

This Appendix details of our implementation of the homotopy algorithm, using the example of the b^p-homotopies that we use in Section 3. The methods used for other homotopies are similar. Our description of the homotopy algorithm follows the description in Besanko, Doraszelski, Kryukov, and Satterthwaite (2010) closely, and our implementation is based on BDK's code, and we use their numerical tolerances.

[32] We have used this tool with both numerical and analytic derivatives, and using different algorithms.

A.2.1 Overview.

An equilibrium for a given set of parameters is defined as the solution to the 4,743 equations presented in Section 2 . We can write these equations collectively as

$$F(\mathbf{x}; b^p, \rho, \sigma) = \mathbf{0}, \tag{A.1}$$

where $\mathbf{x} = (\mathbf{V}^*, \mathbf{V}^{INT*}, \mathbf{p}^*, \lambda^*)$ (i.e., values, for buyers and sellers, and strategies) and we are implicitly conditioning on other parameters that we hold fixed such as the discount factor and the entry cost and scrap value distribution parameters. The objective of a b^p-homotopy is to explore the correspondence

$$F^{-1} = \{\mathbf{x} | F(\mathbf{x}; b^p, \rho, \sigma) = \mathbf{0}, b^p \in [0, 1]\}. \tag{A.2}$$

To follow the correspondence, the homotopy method introduces an ancillary parameter s, so that equation (A.2) becomes,

$$F^{-1} = \{\mathbf{x}(s) | F(\mathbf{x}(s); b^p(s), \rho, \sigma) = \mathbf{0}, b^p \in [0, 1]\}. \tag{A.3}$$

Assuming that a vector $\mathbf{x}$ satisfies the equations, the following conditions must be satisfied for the homotopy to remain on the correspondence

$$\frac{\partial F(\mathbf{x}(s); b^p(s), \rho, \sigma)}{\partial \mathbf{x}} \mathbf{x}'(s) + \frac{\partial F(\mathbf{x}(s); b^p(s), \rho, \sigma)}{\partial b^p} b^{p\prime}(s) = \mathbf{0} \tag{A.4}$$

where $\frac{\partial F(\mathbf{x}(s); b^p(s), \rho, \sigma)}{\partial \mathbf{x}}$ is a (4,743 x 4,743) matrix, $\mathbf{x}'(s)$ and $\frac{\partial F(\mathbf{x}(s); b^p(s), \rho, \sigma)}{\partial b^p}$ are both (4,743 x 1) vectors and $b^{p\prime}(s)$ is a scalar. The solution to these differential equations will have the following form, where $y_i'(s)$ is the derivative of the ith element of $\mathbf{y}(s) = (\mathbf{x}(s), b^p(s))$,

$$y_i'(s) = (-1)^{i+1} \det \left(\left(\frac{\partial F(\mathbf{y}(s); \rho, \sigma)}{\partial \mathbf{y}} \right)_{-i} \right) \tag{A.5}$$

where $_{-i}$ means that the ith column is removed from the (4,743 x 4,744) matrix $\frac{\partial F(\mathbf{y}(s); \rho, \sigma)}{\partial \mathbf{y}}$.

A.2.2 Implementation.

The homotopy procedure is implemented using the FORTRAN routines FIXPNS and STEPNS from HOMPACK90. Jacobians are computed numerically, although we specify which elements of the Jacobian are non-zero.[33,34] The algorithm keeps track of the values of $\mathbf{x}$ and b^p at each step on the path, which we can then use to compute associated outcomes, such as HHI^∞ and P^∞, which we do using the same code as BDK.

Restarting. A practical problem that arises is that a homotopy can stall or start taking an apparently endless sequence of increasingly small steps. We use a few different approaches to try to complete a path. One approach involves running homotopies in the opposite direction (e.g., decreasing b^p, rather than increasing b^p) from equilibria that have already been found. This often connects up sections of a path that have been found using different homotopy runs. If this does not work, we try to identify an adjacent equilibrium by solving the equilibrium equations for a close value of b^p, and then use this value to start a new homotopy path. If this path also does not progress, we solve the equations for additional small changes of b^p.

Computational Burden. The time taken to run a homotopy is usually between one hour and seven hours, when it is run on the University of Maryland's BSWIFT cluster (a moderately sized cluster for the School of Behavioral and Social Sciences).

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[33]STEPNS is a predictor-corrector algorithm where hermetic cubic interpolation is used to guess the next point, and an iterative procedure is then used to return to the path.

[34]For details of the HOMPACK subroutines, please consult manual of the algorithm at `https://users.wpi.edu/~walker/Papers/hompack90,ACM-TOMS_23,1997,514-549.pdf`.

B Classification and Testing for the Existence of SELPM Equilibria

As explained in Section 2, we pay particular attention to one type of non-accommodative equilibria which we call SELPM equilibria.

Definition *A symmetric equilibrium has the "**S**ome **E**xit **L**eads to **P**ermanent **M**onopoly" (SELPM) property if there is some state $e_1^* > 1$, where (i) $\lambda_1(e_1, e_2) = 1$ for all $e_1 \geq e_1^*$*[35] *and $\forall e_2$, including $e_2 = 0$; (ii) $\lambda_2(e_1^*, e_2) < 1$ for some e_2 where $0 < e_2 < e_1^*$, and $\lambda_2(e_1, 0) = 0$ for all $e_1 \geq e_1^*$.*

After some additional discussion of this definition, Appendix B.1 provides a classification of the equilibria identified by the σ- and ρ-homotopies in Sections 4 into accommodative, SELPM and two alternative types of non-accommodative equilibria. Appendix B.2 details the algorithm that we use to identify whether at least one SELPM equilibrium exists.

Discussion. The High and Mid-HHI baseline ($b^p = 0$) equilibria in Table 1 (illustrative parameters) are both SELPM: $e_1^* = 30$ satisfies the definition in both cases. In fact, it is usually the case that $e_1^* = M = 30$ satisfies the definition if an equilibrium is SELPM.[36] Note that our algorithm that tests whether a SELPM equilibrium exists will stop when at the highest e_1 that satisfies the criteria for e_1^*.

Figures B.1 and B.2 provide examples of how play may move through the state space in SELPM equilibria. The first figure shows two paths where we assume that the sellers use the baseline High-HHI equilibrium strategies. The red line shows a path where both sellers make a sale in the first two periods of the game, and the game then evolves to (M, M). The black line shows a path where seller 1 makes the first $M - 4$ sales, and seller 2 then exits. Once seller 1 has made a sale, there is no possibility of entry by a potential entrant seller 2, and the games moves to $(M, 0)$.

[35] Note, that as we are only looking at symmetric equilibria, this condition implies that $\lambda_2(e_1, e_2) = 1$ for all $e_2 \geq e_1^*$.

[36] Intuitively, a laggard will have the strongest incentive to exit, and a potential entrant the least incentive to enter, when it is as far behind the leader as possible.

Figure B.1: Baseline High-HHI Equilibrium: Examples of Possible Paths Through the State Space For a Game Starting at (1,1). The numbers in each cell are seller 2's continuation probabilities. The circular arrows indicate no sale being made, due to the buyer choosing the "outside option".

$\mathbf{e_1}$

$\mathbf{e_2}$

	out	1	2	...	$M-4$	$M-3$	$M-2$	$M-1$	M
out	0.9583	0	0		0	0	0	0	0
1	1	0.9996	0.7799		0.7777	0.7777	0.7777	0.7777	0.7777
2	1	1	1		1	1	1	1	1
...									
$M-4$	1	1	1		1	1	1	1	1
$M-3$	1	1	1		1	1	1	1	1
$M-2$	1	1	1		1	1	1	1	1
$M-1$	1	1	1		1	1	1	1	1
M	1	1	1		1	1	1	1	1

Figure B.2 provides a second (hypothetical) example where a potential entrant seller 2 could enter in some states. However, $e_1 = 30$ satisfies the definition of e_1^*, so the equilibrium is SELPM.

The following are examples of strategies where the equilibrium would not be SELPM:

1. accommodative equilibria (i.e., $\lambda_2(e_1, e_2) = 1$ for all $e_1, e_2 \geq 1$);

2. an equilibrium where $\lambda_2(M, 0) > 0$ (for example, due to a low lower bound on entry costs);

3. an equilibrium where $\lambda_1(M, M) < 1$ (for example, due to a high upper bound on scrap values and/or intense duopoly competition); or,

4. an equilibrium where $\lambda_2(M, 0) = 0$, $\lambda_1(M, e_2) = 1$ for $e_2 \geq 2$ or $e_2 = 0$, but $\lambda_1(M, 1) < 1$. If the state reaches $(30, 2)$ the game will either proceed to (M, M) (permanent

Figure B.2: Alternative SELPM Example with Possible Re-entry: Examples of Possible Paths Through the State Space For a Game Starting at (1,1). The numbers in each cell are seller 2's continuation probabilities. The circular arrows indicate no sale being made, due to the buyer choosing the "outside option".

$\mathbf{e_1}$

		out	1	2	...	$M-4$	$M-3$	$M-2$	$M-1$	M
	out	0.958	0.758	0.643		0.123	0.042	0	0	0
	1	1	1	0.945		0.689	0.675	0.672	0.669	0.666
	2	1	1	1		0.695	0.689	0.686	0.683	0.681
	...									
$\mathbf{e_2}$	$M-4$	1	1	1		1	1	1	1	1
	$M-3$	1	1	1		1	1	1	1	1
	$M-2$	1	1	1		1	1	1	1	1
	$M-1$	1	1	1		1	1	1	1	1
	M	1	1	1		1	1	1	1	1

duopoly) with no exit, or seller 2 may exit and seller 1 will be a permanent monopolist. However, it fails to meet our definition because seller 1 may exit in state $(M, 1)$. We require the condition that $\lambda_1(e_1, e_2)$ in all $e_1 \geq e_1^*$ because it allows us to construct an algorithm that can check whether a SELPM equilibria exists or not under weaker assumptions on the scale of problem for which we can find all equilibria. Of course, if this equilibrium exists, it is also possible that a different equilibrium, where $\lambda_1(30, 1) = 1$, will satisfy the SELPM definition.

B.1 Classification.

We now classify the equilibria identified in Section 4 into different types. Two mutually exclusive types are accommodative (see definition in Section 2) and SELPM. We also consider two other types:

Definition *An equilibrium has the "**A**ny **E**xit **L**eads to **P**ermanent **M**onopoly" (AELPM) property if (i)* $\lambda_1(\mathbf{e}) = 1$ *for all* $\mathbf{e} = (e_1, e_2)$ *where* $e_1 \geq e_2$*; (ii) there is some* $\mathbf{e} = (e_1, e_2)$ *where* $e_1 > e_2 > 0$ *and* $\lambda_2(\mathbf{e}) < 1$*, and (iii) for any* $\mathbf{e} = (e_1, e_2)$ *where* $e_1 > e_2 > 0$ *and* $\lambda_2(\mathbf{e}) < 1$*,* $\lambda_2(e_1', 0) = 0$ *for* $e_1' \geq e_1$*.*

In an AELPM equilibrium, the only exit from duopoly will be by a strict laggard and there will be no re-entry once a laggard exits. Any AELPM equilibrium will be SELPM.[37] But, SELPM equilibria may not be AELPM. For example, the High-HHI baseline equilibrium in Table 1 is not AELPM because there is a chance that sellers exit in state (1,1), so that there is a small probability that both sellers exit, in which case there may be re-entry.

We also consider equilibria that satisfy BDK's definition of "aggressive" equilibria.

Definition *An equilibrium is "aggressive" if* $p_1(\mathbf{e}) < p_1(e_1, e_2+1), p_2(\mathbf{e}) < p_2(e_1, e_2+1)$*, and* $\lambda_2(\mathbf{e}) < \lambda_2(e_1, e_2 + 1)$ *for some state* $\mathbf{e} = (e_1, e_2)$ $e_1 > e_2 > 0$*.*

This definition depends on both prices and continuation strategies. Aggressive equilibria are not accommodative, and they may or may not be AELPM or SELPM.

B.1.1 Classification for σ- and ρ-Homtopies.

Figure B.3(a) and (b) shows a classification of the equilibria found by σ- and ρ-homotopies for different values of b^p. The other parameters are held at their baseline values. The HHI^{∞} is shown on the y-axis. The different line styles indicate the different type of equilibria. Recall that all AELPM equilibria are SELPM. For these parameters we find that:

- all identified equilibria are either accommodative or SELPM (i.e., this classification is exhaustive for the equilibria that the homotopies identify for these parameters); and

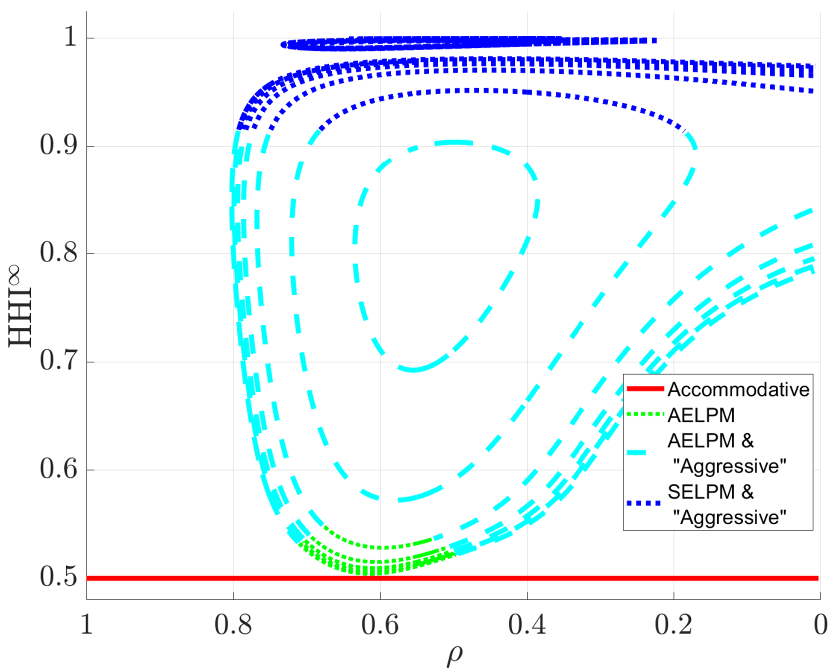
[37] In particular, the fact that a leader will never exit rules out the fourth type of non-SELPM equilibrium above. Any value of e_1 where there is some possibility that a laggard seller 2 exits will meet the e_1^* definition.

- all identified aggressive equilibria are SELPM, although many SELPM equilibria are not aggressive.

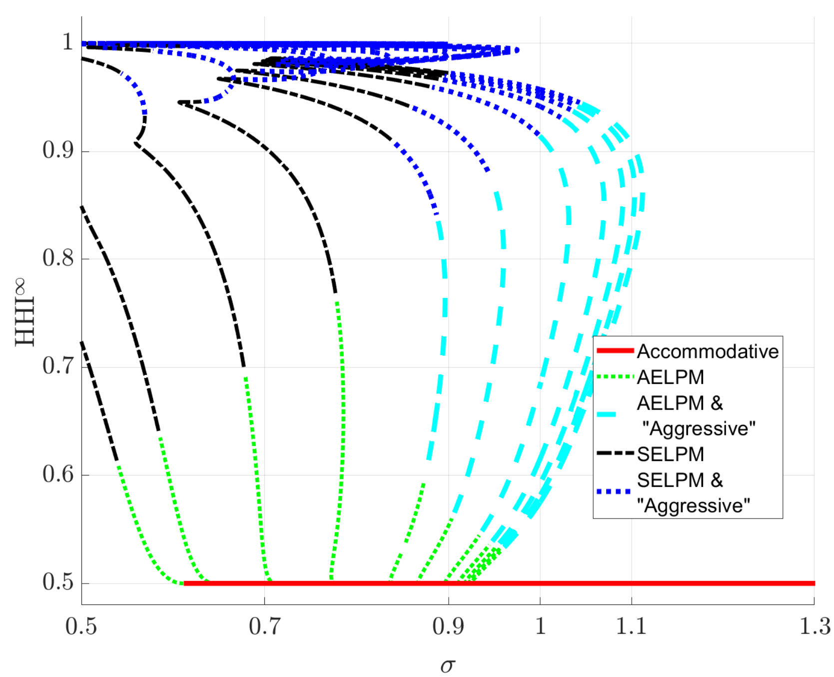
There is also an interesting pattern where the AELPM equilibria tend not to be the equilibria with the highest implied values of HHI^{∞}. Even though high HHI^{∞} equilibria tend to have low duopoly prices, and it is not attractive for a potential entrant to enter against a monopolist, there is usually some probability of exit in symmetric duopoly states, particularly (1,1), so these equilibria do not meet the AELPM criteria.

Figure B.3: Classification of Equilibria Identified by ρ- and σ-Homotopies for Various b^p. Other parameters at their illustrative values. See text Figures 4(a) and (c) for which line corresponds to which b^p. Equilibria indicated as "SELPM" or "AELPM" only do not satisfy the definition of aggressive equilibria.

(a) ρ-Homotopy Paths ($\sigma = 1$)



(b) σ-Homotopy Paths ($\rho = 0.75$)



B.2 An Algorithm for Testing if SELPM Equilibria Exist

A property of SELPM equilibria is that once the state e_1^* has been reached, state transitions have the directional property that the state will evolve to (M, M) or $(M, 0)$ without returning to a previously visited state. As discussed by Iskhakov, Rust, and Schjerning (2016), recursive algorithms, which solve for equilibria in a sequence of individual states, can be used when states evolve directionally. However, the way that we use this idea is novel in at least two ways. First, we consider a directional property that applies to a certain type of equilibrium in part of the state space, rather than a property which has to apply to all equilibria given primitives of the model. Second, we apply a recursive algorithm to find whether this type of equilibrium exists, rather than trying to find all equilibria.

We proceed as follows. First, we describe how the algorithm proceeds through the state space, and how it terminates in success or failure, without providing details of how we solve for equilibrium strategies in any particular state. Instead, we make assumptions about our ability to solve for all equilibria in a particular state given continuation values if the state changes.[38] Second, we provide the proof that, under these assumptions, our algorithm will terminate in success if and only if a SELPM equilibrium exists. Finally, we detail the mechanics of how we solve for equilibria in different types of states.

B.2.1 Overview of the Algorithm

The algorithm recursively solves for equilibrium strategies in each state until we either (i) find an equilibrium path where there is an e_1 state that meets the SELPM definition of e_1^* ("success"), or (ii) find that all paths are inconsistent with SELPM ("failure"). Notably, either outcome may be achieved by going only through a small part of the state space.

Figure B.4 describes the recursive path that the algorithm takes through the state space. The key feature is that we only construct and follow paths that are consistent with SELPM in states $e_1 \geq e_1^*$. For example, this implies that if the industry becomes a monopoly then it will remain so. We therefore solve for equilibrium prices and values in duopoly states

[38] As noted by Iskhakov, Rust, and Schjerning (2016), assumptions are needed as no algorithms are guaranteed to find all equilibria in particular states, outside of some special cases that do not apply here. However, we explain why we are confident that, in practice, we are able to find all equilibria.

Figure B.4: Outline of the Recursive Algorithm.

Preliminaries:

Create matrix E that will contain strategies and values on the equilibrium path.

For all states where firm 2 is a potential entrant (i.e., $(e_1,0)$ for all $e_1>0$), solve for equilibrium strategies and values assuming that $\lambda_1(e_1,0)=1$ and $\lambda_2(e_1,0)=0$ for all e_1 and store in E.

Solve for equilibrium strategies and values in state (30,30) assuming that $\lambda_1(30,30)=\lambda_2(30,30)=1$. This will correspond to the static Nash price. If implied $\beta V_1^S(30,30) < (\bar{X}+\Delta X)$, then there is no SELPM equilibrium, so can stop. Otherwise add these strategies to E.

Set e1==30, e2==29, call [fail,success] = recursion_function(e1,e2,E)

If success==1, there is a SELPM equilibrium.

If fail==1, there is no SELPM equilibrium.

Recursive Function

```
function [fail,success] = recursion_function(e1start,e2start,E)

success=0
fail=0

e1=e1start
e2=e2start

while success==0 && fail==0 && e1>=1,

   e2=e2start
   while success==0 && fail==0 && e2>=0,

      % STATE WHERE LAGGARD IS A POTENTIAL ENTRANT
      if e2==0,
            %Check whether firm 2 would want to enter in state ($e_1$,0) given
            %values currently in E, i.e., $\lambda_2(e_1,0)=0$
            if $\beta V_2^S(e_1,1) > (\bar{S}-\Delta S)$
                  fail=1
            else
                  %Check whether firm 1 will want to continue with
            probability 1
                  if $\beta V_1^S(e_1,0) < (\bar{X}+\Delta X)$
                        fail=1
                  else
                        if E implies some follower exit in a state where
                  leader state is $e_1$,
                              success=1
                        end
                  end
                  record the prices and values for ($e_1$,0) in E
            end
      else
            % STATE WHERE INCUMBENT FIRMS ARE SYMMETRIC
            if e1==e2,
                  find *assumed-to-be unique* state-specific symmetric prices
                  and values equilibrium in ($e_1$, $e_1$), using continuation
                  values from E, assuming $\lambda_2(e_1,e_1)=\lambda_1(e_1,e_1)=1$.
```

Figure B.4: Outline of the Recursive Algorithm cont.

```
            %Verify leader will continue with probability 1.
            if βV_1^S(e_1,e_1) < (X̄ + ΔX)
                  fail=1
            else
                  record the prices and values for (e1, e1) in E
            end
            if e1==1 && no exit from any e1 states, % accommodative
            equilibrium
                  fail=1;
            end
      % STATE WHERE THE FIRM 1 IS A LEADER, 2 IS ACTIVE LAGGARD
      elseif e1>e2,
            find all pricing and seller 2 continuation probability
            state-specific equilibria for (e1,e2), using continuation
            values from the current top row of E, when assume
            λ1(e1,e2)=1.

            if a unique equilibrium,
                  if βV_1^S(e_1,e_2) < (X̄ + ΔX),
                        fail=1
                  else
                        record the prices and values for (e1,e2) in E
                  end
            end

            if multiple equilibria,
                  neq = number of equilibria

                  identify the equilibria where λ2(e1,e2)<1-(e-10).
                  sort these equilibria in reverse λ2(e1,e2) order,
                  except if no exit yet identified for e1 states, in
                  which case place the equilibrium with the highest
                  λ2(e1,e2)<1 first.

                  local_fail=1;
                  eq=1

                  while local_fail==1 && success==0 && eq<=neq,
                        add E'=E with equilibrium(eq) added,
                        e2'=e2-1;
                        [local_fail,success]=
                        recursion_function(e1,e2',E')

                        eq=eq+1
                  end
            end
            fail=local_fail
      end
   end
   e2=e2-1
 end
 e1=e1-1;e2start=e1
end
```

assuming that this will be what happens in monopoly states, before verifying that, in fact, potential entrants would not want to enter. The SELPM-consistent equilibrium strategies and values on the current path (including for monopoly states that the algorithm has not yet reached) are stored in a set of matrices, that, for ease of description, we collective label as E.[39]

To understand the process, consider the illustrative parameters with $b^p = 0$. The algorithm solves for equilibria in monopoly states when seller 1 and the buyer assumes that the potential entrant will not enter. Consistent with Table 1, this implies the incumbent will set prices of 8.54 in state (30,0), and for example, 8.72 in state (2,0), although this price will only be relevant if the search for an e_1^* continues back to $e_1 = 2$. It then solves for the SELPM-consistent equilibrium in state (30,30), where neither seller will exit, before progressing through the states (30,29), (30,28),...,(30,2), using the continuation values in the states that the game could move to in a SELPM equilibrium (including (30,0)) in order to solve the game in a particular state. In these states, we find that the only SELPM-consistent equilibria have $\lambda_2 = 1$. In state (30,1) we find three equilibria with $\lambda_2 = 0.7777$, 0.9577 and 1. The algorithm selects the 0.9577 (Mid-HHI) equilibrium to try first. In this case, it only needs to check if $\lambda_1(30,0) = 1$ and $\lambda_2(30,0) = 0$ given the implied $V_1^S(30,1)$ and $V_2^S(30,1)$. Both checks are passed so the criteria for e_1^* are satisfied by $e_1 = 30$ and the algorithm terminates in success. If, counterfactually, we had found multiple equilibria in state (30,2), then algorithm would have selected one path, extended that path to find an equilibrium in state (30,1) and then performed the check on continuation probabilities in state (30,0). If the SELPM conditions are rejected on one path, the next path, if one is available, is chosen.

B.2.2 Properties of the Algorithm

We make two claims about the property of the algorithm.

Claim 1 *If the algorithm terminates in success, then a SELPM equilibrium exists.*

Proof. Inspection reveals that if the algorithm terminates in success for $e_1 = e_1'$ then (i)

[39]Our code also assumes that seller 2 is the leader, rather than seller 1. We present our description with seller 1 as the leader as it is easier to follow.

$\lambda_1(e_1, e_2) = 1$ for all $e_1 \geq e_1'$ and all e_2, including $e_2 = 0$[40], and (ii) $\lambda_2(e_1', e_2) < 1$ for some $0 < e_2 < e_1'$ and $\lambda_2(e_1, 0) = 0$ for all $e_1 \geq e_1'$.

Therefore, the path that terminates in success has equilibrium strategies and values consistent with SELPM for all states where $e_1 \geq e_1'$, and e_1' satisfies the criteria for e_1^* in the definition.[41]

It remains to show that a set of equilibrium strategies and values in earlier states must exist that, when combined with these strategies and values, would form an equilibrium in the whole game. In a SELPM equilibrium, once state e_1^* has been reached, play will only move through states where equilibrium strategies and values have been calculated by the algorithm. Therefore, we only require that an equilibrium exists in a reduced game where the states are $e_1 = 1, ..., e_1^* - 1$ and the terminal payoffs of players if a buyer purchases from seller 1 in a state $(e_1^* - 1, e_2)$ are $V_i^{S,INT}(e_1^*, e_2)$ and $V^{B,INT}(e_1^*, e_2)$. Existence of an equilibrium in this reduced game follows from the arguments in Doraszelski and Satterthwaite (2010). ■

To prove that the algorithm will terminate in success if a SELPM equilibrium exists, we make three additional assumptions.

Assumption 1 *There is a unique state-specific equilibrium (i.e., values of p_1, V_1^S, $V_1^{S,INT}$, V^B, $V^{B,INT}$ satisfying the monopoly state version of the equilibrium equations in Section 2) in a monopoly state $(e_1, 0)$ with $e_1 < M$, given fixed buyer and seller continuation values if the buyer purchases from seller 1, if $\lambda_1(e_1, 0) = 1$ and $\lambda_2(e_1, 0) = 0$.*

While we are making an assumption for states $(e_1, 0)$ with $e_1 < M$, the existence of only one SELPM-consistent equilibrium in monopoly state $(M, 0)$ follows from the fact that this must be an absorbing state, and that the equilibrium will involve static Nash pricing by the monopolist for all values of b^p. We refer to this as Property 1 below.

Assumption 2 *There is a unique symmetric state-specific equilibrium (i.e., values of p_1, p_2, V_1^S, V_2^S, $V_1^{S,INT}$, $V_2^{S,INT}$, V^B, $V^{B,INT}$ satisfying the duopoly state equations in Section 2)*

[40] One may notice that the algorithm does not solve for strategies in a state where seller 2 is the leader, e.g., (29,30). However, under the restriction that we are only solving for symmetric equilibria, then for the algorithm to be looping through e_2 states for $e_1 = 29$ it must be the case that $\lambda_2(30, 29) = 1$ on the path that is being tracked, so it follows that $\lambda_1(29, 30) = 1$.

[41] Of course, the strategies found in monopoly states where $e_1 < e_1'$ may not be consistent with equilibrium behavior, but they would have been consistent if, in search of an e_1 state meeting the SELPM-criteria, the algorithm had visited these states.

in a symmetric duopoly state (e_1, e_1) *with* $e_1 < M$*, given fixed buyer and seller continuation values if the buyer purchases from sellers 1 or 2, when* $\lambda_1(e_1, e_1) = \lambda_2(e_1, e_1) = 1$.

While we are making an assumption for states (e_1, e_1) with $e_1 < M$, the existence of only one SELPM-consistent equilibrium in duopoly state (M, M) follows from the fact that this must be an absorbing state, and that the equilibrium will involve static Nash pricing by each seller for all values of b^p. The form of demand implies that there is a unique static Nash equilibrium (Caplin and Nalebuff (1991), Mizuno (2003)). We refer to this as Property 2 below.

As noted below, we have never found examples under which either of these assumptions is violated.

Assumption 3 *We are able to find all state-specific equilibria (i.e., values of* p_1*,* p_2*,* λ_2*,* V_1^S*,* V_2^S*,* $V_1^{S,INT}$*,* $V_2^{S,INT}$*,* V^B*,* $V^{B,INT}$ *satisfying the duopoly state version of the equations in Section 2) in an asymmetric duopoly state* (e_1, e_2) *with* $e_1 > e_2$*, given fixed buyer and seller continuation values if the buyer purchases from seller 2 or seller 1 (if* $e_1 < M$*), when* $\lambda_1(e_1, e_2) = 1$.

We detail below the procedure that we use, which is based on the assumption that there is a unique pricing and value equilibrium when we fix the value of $\lambda_2(e_1, e_2)$, and that this equilibrium is continuous in $\lambda_2(e_1, e_2)$. We explain why we believe this assumption holds below, although, like BDK, we find that it can be challenging to find any equilibrium for low values of σ, and for this reason we do not report results for $\sigma < 0.5$.

Claim 2 *Under assumptions 1-3, if a SELPM equilibrium exists, then our algorithm will terminate in success.*

Proof. The assumptions and Properties 1 and 2 imply that the algorithm will follow, and evaluate, every possible SELPM-consistent equilibrium path before terminating in failure. Therefore if a SELPM-consistent state e_1^* exists, the algorithm will find it. ■

B.3 Methods for Solving for Equilibria in Specific States

We now describe how we solve for equilibria that are consistent with SELPM in specific states. We describe our routines assuming that $\sigma = 1$ to reduce notation. Our examples assume the illustrative parameters, with $\rho = 0.75$ and $\sigma = 1$, unless otherwise stated.

B.3.1 Solving for Equilibria in Monopoly States $(e_1, 0)$ assuming $\lambda_1(e_1, 0) = 1$ and $\lambda_2(e_1, 0) = 0$.

Consider a state $(e_1 < M, 0)$. Assuming $\lambda_1(e_1, 0) = 1$ and $\lambda_2(e_1, 0) = 0$, the following equations determine the equilibrium values of V^B, $V^{B,INT}$, V_1^S,$V_1^{S,INT}$ and p_1 where seller 1's marginal cost is c,

$$V^B = b^p \ln(\exp(V^{B,INT}) + \exp(v - p_1 + V^{B,INT}(e_1 + 1, 0)))+ \tag{B.1}$$
$$(1 - b^p)(D_1 V^{B,INT}(e_1 + 1, 0) + (1 - D_1)V^{B,INT})$$
$$V_1^S = (p_1 - c + V_1^{S,INT}(e_1 + 1, 0))D_1 + V_1^{S,INT}(1 - D_1) \tag{B.2}$$
$$D_1 + (p_1 - c + V_1^{S,INT}(e_1 + 1, 0) - V_1^{S,INT})\frac{\partial D_1}{\partial p_1} = 0 \tag{B.3}$$
$$V^{B,INT} = \beta V^B \tag{B.4}$$
$$V_1^{S,INT} = \beta V_1^S \tag{B.5}$$

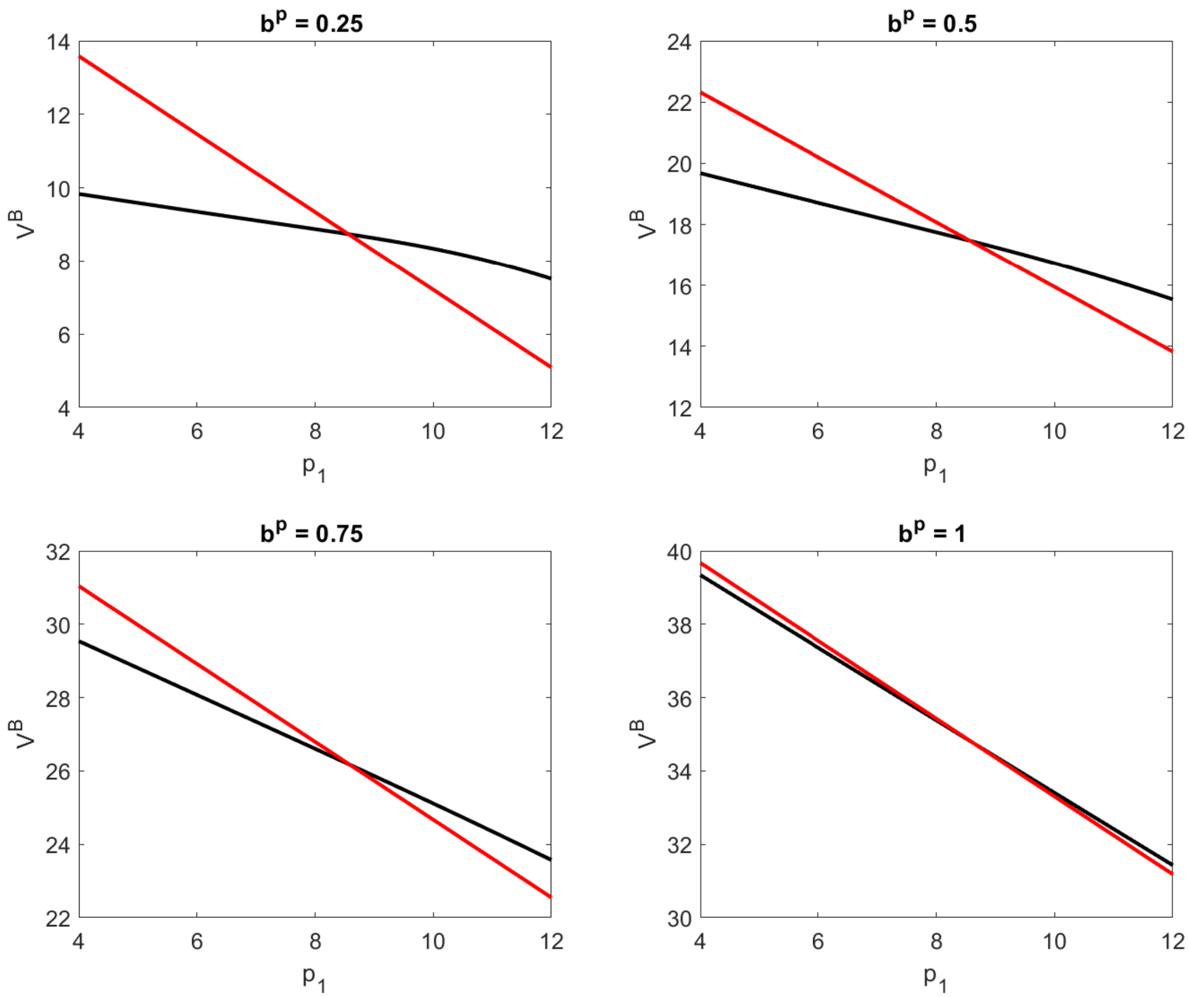
where $D_1 = \frac{\exp\left(v_1 - p_1 + V^{B,INT}(e_1+1,0)\right)}{\exp(v_1 - p_1 + V^{B,INT}(e_1+1,0)) + \exp(V^{B,INT})}$ assuming, following BDK, that $v_0 = p_0$.

We could solve these sets of equations recursively for different monopoly states. However, we find it quicker to solve the equations for all of the monopoly states simultaneously in MATLAB using `fsolve`. We also reduce the number of variables by solving for V^B, V_1^S and p_1 and using these values to solve for $V^{B,INT}$ and $V_1^{S,INT}$ as needed.

Discussion of the Uniqueness Assumption. We have performed an analysis to check whether Assumption 1 is likely satisfied. Specifically, we can look at whether two equilibrium curves intersect more than once. The first curve solves the value of V^B as a function of p_1, reflecting equation (B.1). The second curve solves for the value of p_1 that maximizes the seller's value, given V^B, as determined by the first-order condition (B.3).

Figure B.5 presents examples of what these curves look like for state $(10, 0)$ using the

Figure B.5: Monopoly State Equations in State $(10, 0)$: black curve is the value of V^B as a function of p_1, red curve is the optimal p_1 given V^B. There is an equilibrium where the lines intersect.



illustrative parameters when $b^p = 0.25$, $0.5, 0.75$ and 1. The black curves denote the value of V^B given p_1, and the red curves reflect the value-maximizing choices of p_1 given values of V^B. The curves cross only once in every case, consistent with a single equilibrium. We have verified that there is only one intersection for a very large number of different values of ρ, σ, b^p, $V^S(e_1+1,0)$ and $V^B(e_1+1,0)$.[42]

B.3.2 Solving for Equilibrium in Absorbing Duopoly State (M, M).

(M, M) is an absorbing state in a SELPM equilibrium. This implies that there is a unique SELPM-consistent equilibrium where prices are the same as static Nash prices with non-strategic buyers (uniqueness of these prices follows from the multinomial logit form of demand (e.g., Mizuno (2003))).

We find equilibrium prices by solving static pricing first-order conditions,

$$D_i + (p_i - c)\frac{\partial D_i}{\partial p_i} = 0,$$

and then calculating the implied buyer and seller values (V^S). We verify that βV^S is greater than the maximum possible scrap value, so that exit is not optimal. If exit could be optimal, there is no SELPM equilibrium.

B.3.3 Solving for Equilibria in Other Duopoly States $(e_1, e_2), e_1 \geq e_2 > 0, e_2 < M$.

In a duopoly state we want to solve for all SELPM-consistent values of

- prices (p_1, p_2)
- values $(V_1^S, V_2^S, V_1^{S,INT}, V_2^{S,INT}, V^B, V^{B,INT})$
- continuation probability for seller 2 (λ_2), although SELPM implies that $\lambda_2 = 1$ if $e_1 = e_2$ for $e_1 \geq e_1^*$.

[42]Specifically, we use b^p values on a grid [0.2,0.4,0.6,0.8,1], ρ values [0,0.1,0.2,..,0.9,1], σ values [0.5,0.6,..,1.1,1.2], $V^S(e_1+1,0)$ values [60,65,..,95,100] and $V^B(e_1+1,0)$ values b^p*[20,25,30,35,40]. This gives a total of 19,800 combinations that we check. We have also experimented with other values.

The continuation probability for seller 1 must be 1. The nine variables must satisfy the following nine equations

$$V_i^S - D_i(p_1, p_2, V^B)(p_i - c_i(e_i)) - \sum_{k=0,1,2} D_k(p_1, p_2, V^B) V_i^{S,INT}(\mathbf{e}'_k) = 0 \text{ for } i = 1, 2 \quad \text{(B.6)}$$

where $V_i^{S,INT}(\mathbf{e}'_0) = V_i^{S,INT}$,

$$V_1^{S,INT} = \beta\left(\lambda_2 V_1^S + (1-\lambda_2)V_1^S(e_1, 0)\right) \text{ and } V_2^{S,INT} = \beta\lambda_2 V_2^S + (1-\lambda_2)E(X|\lambda_2) \quad \text{(B.7)}$$

$$D_i(p_1, p_2, V^B) + \sum_{k=0,1,2} \frac{\partial D_k(p_1, p_2, V^B)}{\partial p_i} V_i^{S,INT}(\mathbf{e}'_k) + (p_i - c_i(e_i)) \frac{\partial D_i(p_1, p_2, V^B)}{\partial p_i} = 0 \text{ for } i = 1, 2 \quad \text{(B.8)}$$

$$\lambda_2 - F_{\text{scrap}}(\beta V_2^S) = 0 \quad \text{(B.9)}$$

$$V^B = b^p \log\left(\sum_{k=0,1,2} \exp\left(v_k - p_k + V^{B,INT}(\mathbf{e}'_k)\right)\right) - (1-b^p) \sum_{k=0,1,2} D_k(p_1, p_2, V^B) V^{B,INT}(\mathbf{e}'_k), \quad \text{(B.10)}$$

where $V^{B,INT}(\mathbf{e}'_0) = V^{B,INT}$,

$$V^{B,INT} = \beta\left(\lambda_2 V^B + (1-\lambda_2)V^B(e_1, 0)\right), \quad \text{(B.11)}$$

where $\mathbf{e}'_k$ is the state that the game transitions to when the buyer purchases from k, and $E(X|\lambda_2)$ is the expected scrap value when seller 2 exits with probability $1-\lambda_2$. When this involves a change of state, we take the continuation values as given. For example, if $e_1 < M$[43],

$$V_1^{S,INT}(\mathbf{e}'_1) = \beta\left(\lambda_2(e_1+1, e_2)V_1^S(e_1+1, e_2) + (1-\lambda_2(e_1+1, e_2))V_1^S(e_1+1, 0)\right) \quad \text{(B.12)}$$

$$V_2^{S,INT}(\mathbf{e}'_1) = \beta\left(\lambda_2(e_1+1, e_2)V_2^S(e_1+1, e_2) + (1-\lambda_2(e_1+1, e_2))E(X|\lambda_2(e_1+1, e_2))\right), \quad \text{(B.13)}$$

where $E(X|\lambda_2(e_1+1, e_2))$ is the expected scrap value if seller 2 exits with probability $1-$

[43] Alternatively, if $e_1 = M$, $V_1^{S,INT}(\mathbf{e}'_1) = \beta\left(\lambda_2 V_1^S + (1-\lambda_2)V_1^S(M, 0)\right)$ and $V_2^{S,INT}(\mathbf{e}'_1) = \beta\left(\lambda_2 V_2^S + (1-\lambda_2)E(X|\lambda_2)\right)$ and $V^{B,INT}(\mathbf{e}'_1) = \beta\left(\lambda_2 V^B + (1-\lambda_2)V_2^B(e_1, 0)\right)$, so they depend on the endogenous λ_2, V^B and V_1^S, because a sale by seller 1 does not change the state.

$\lambda_2(e_1+1,e_2)$.

$$V^{B,INT}(\mathbf{e}_2') = \beta\left(\lambda_2(e_1,e_2+1)V^B(e_1,e_2+1)+(1-\lambda_2(e_1,e_2+1))V^B(e_1,0)\right), \quad \text{(B.14)}$$

$$V^{B,INT}(\mathbf{e}_1') = \beta\left(\lambda_2(e_1+1,e_2)V^B(e_1+1,e_2)+(1-\lambda_2(e_1+1,e_2))V^B(e_1+1,0)\right) \quad \text{(B.15)}$$

If $e_1 = e_2 \geq e_1^*$ then a SELPM-consistent equilibrium must have $\lambda_2 = \lambda_1 = 1$. Therefore, for these states, we solve for equilibrium prices and values assuming that $\lambda_2 = 1$, and then we verify that the solution implies that βV_2^S is greater than the highest possible scrap value, implying that $\lambda_2 = 1$ is optimal. In practice, we solve for V_i^S, V^B and p_i for $i = 1, 2$, substituting in for $V_i^{S,INT}$ and $V^{B,INT}$.

If $e_1 > e_2 \geq e_1^*$ then a SELPM-consistent equilibrium may have $\lambda_2 < 1$, and we may find multiple equilibria. Our method for identifying the set of SELPM-consistent equilibria assumes that there is a unique equilibrium for a given value of λ_2.[44] We specify a grid of values of λ_2, with steps of 0.01, and for each of these values we solve the equations (B.6), (B.8) and (B.10) for p_i, V_i^S and V^B, substituting into equations (B.7) and (B.11) for the values of $V_i^{S,INT}$ and $V^{B,INT}$.[45] We then calculate the best response value of λ_2, $\lambda_2^{\mathrm{BR}}(\lambda_2)$, given V_2^S using equation (B.9).

Figure B.6 shows examples of the function $\lambda_2^{\mathrm{BR}}(\lambda_2)$ for the illustrative parameters, for states (30,1) and (30,5), with $b^p = 0$ and $b^p = 0.2$. There are equilibria at the points where the functions cross the 45-degree line. We find the precise intersection using locations between gridpoints either side of an intersection as starting points, before verifying that the solution is consistent with the leader continuing with probability 1, as required for SELPM.[46,47]

Discussion of the Uniqueness Assumption. As noted, our approach assumes that there

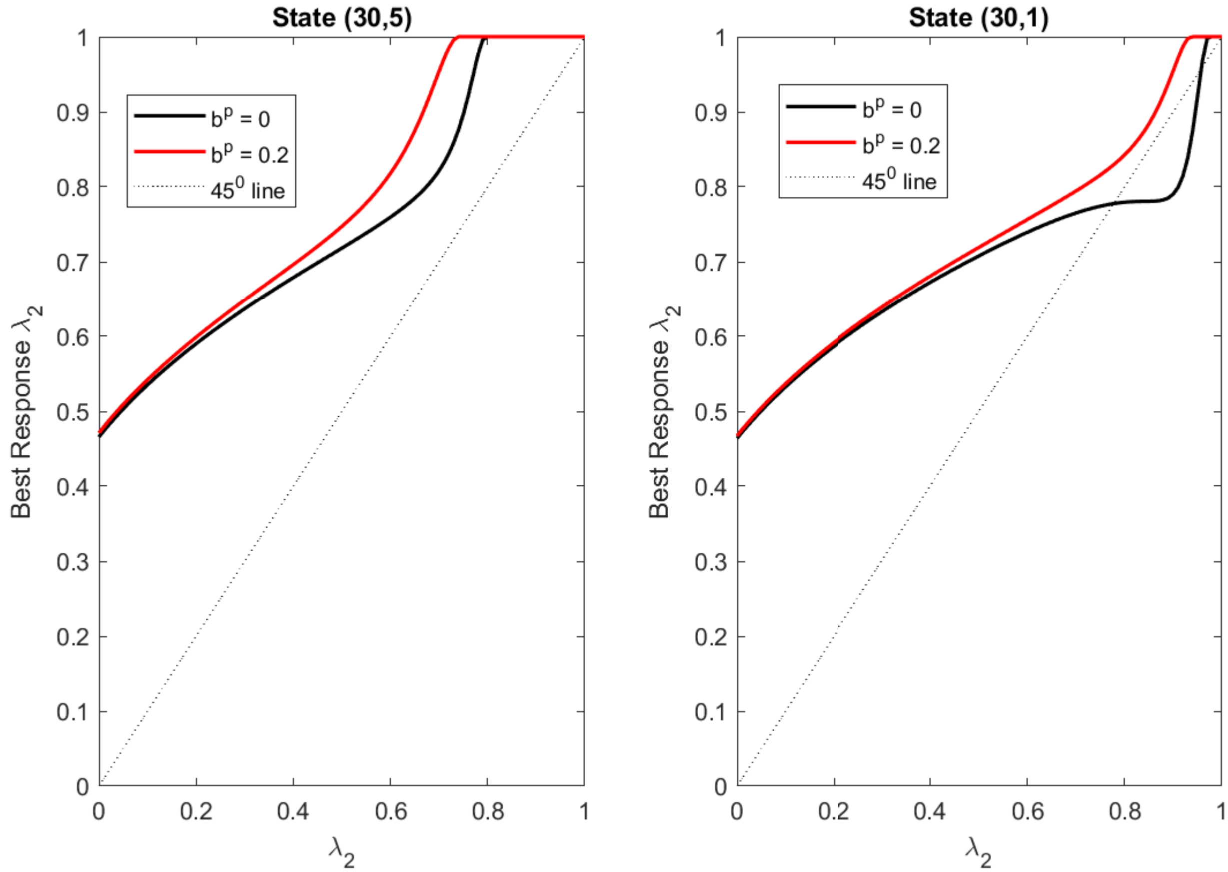
[44]Given that equilibrium prices directly affect V_2^S and λ_2 is a strictly increasing function of V_2^S for $\lambda_2 < 1$, we regard this assumption as weak for $\lambda_2 < 1$.

[45]Occasionally the equations do not solve using the starting values chosen, in which case we use a Pakes-McGuire type of routine to find alternative starting values.

[46]We initially try to find the intersection by starting at the neighboring gridpoints, but if this fails, we use convex combinations of the gridpoints as starting values until the intersection is identified.

[47]As the figure suggests, it is possible that we would miss an intersection where the function is close to forming a tangent with the 45-degree line. We have found that gridpoints of 0.01 are adequate to identify whether SELPM equilibria exist, in the sense that our results do not change if we use a finer grid. This is partly because even if we do just miss an intersection in one particular state (e_1, e_2), there will often be a clearer intersection for state $(e_1, e_2 - 1)$ that we will capture, which may allow us to show that a SELPM equilibrium exists.

Figure B.6: Best Response Continuation Probability Functions for Seller 2 Given Endogenous Pricing Choices by Both Sellers. Intersections with the 45^0-degree line are equilibria.



is a unique pricing equilibrium given an assumed value of λ_2 when $\lambda_1 = 1$. There are two types of evidence that support this presumption. First, we have never identified an instance of multiple equilibria for any of the parameters that we have considered, even when using multiple different starting points or alternative solution algorithms. Second, we have investigated whether there could be multiple equilibria by using a reaction function-type of analysis.

Specifically, for a given value of λ_2 and the continuation values, we solve the equations for V^B, V_1^S and the first-order condition for p_1 for a grid of alternative values of p_2. We then solve the equations for V^B, V_2^S and the first-order condition for p_2 for a grid of alternative values of p_1. We can then draw curves $p_1^*(p_2)$ and $p_2^*(p_1)$, which reflect optimal behavior of buyers and the other seller to the assumed price. The intersections correspond to equilibria, and we can test whether they intersect more than once. Figures B.7 presents some examples of these curves for the illustrative parameters, $b^p = 0$ or $b^p = 1$ and $e_1 = 30$ and $e_2 = 1$.

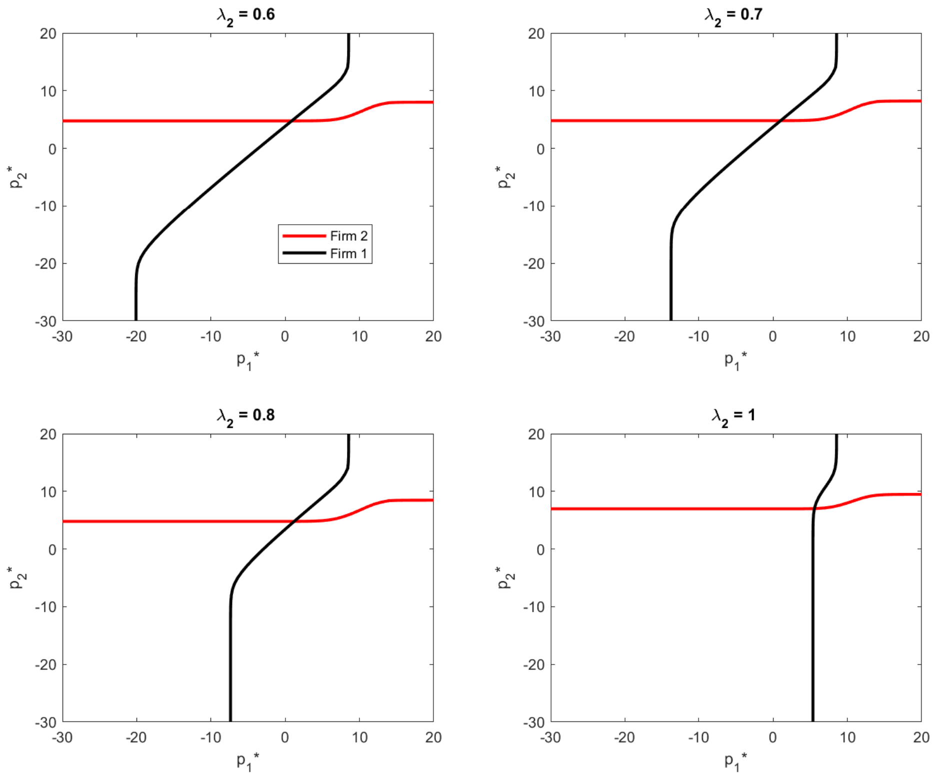
Recall that in the state (30,1), if the buyer purchases from seller 1, the state remains (30,1), whereas if seller 2 makes a sale, the state transitions to (30,2), where, for these parameters, there is always a unique equilibrium. If seller 2 is setting a much lower price in state (30,1) than in state (30,2), a strategic buyer will have an incentive to shift demand towards seller 1 in order to keep the state the same in future periods. As a result, seller 1's optimal price is less sensitive to seller 2's price in this state when $b^p = 1$, which accounts for the change in the slope of the reaction functions. However, in all cases, the reaction functions only intersect once, and there is a single equilibrium.[48]

In practice, it is prohibitive to perform this check for all values of λ_2 for all states for all parameters. However, our checking algorithm does perform this check in states where $e_1 = M$ for λ_2 =0.55, 0.65, 0.75, 0.85 and 0.95. We have never found parameters where there is ever more than one intersection. This is also the case when we have solved games for many different sets of arbitrary continuation values and parameters.

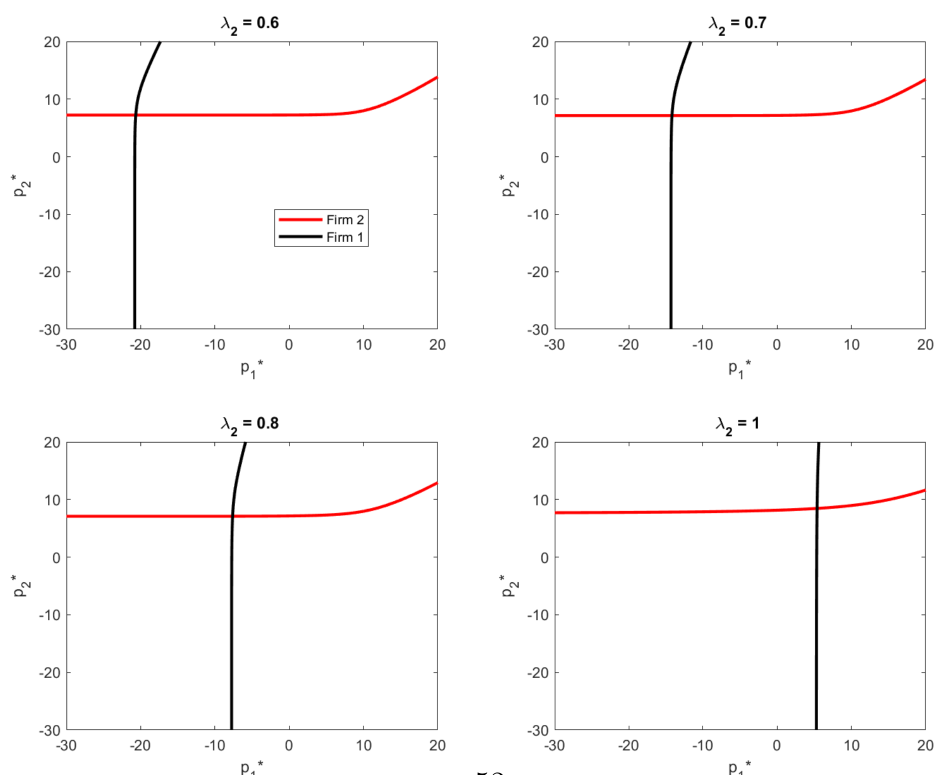
[48]Note that in a state (e_1, e_2) where $e_1 < M$, the buyer cannot keep the state the same by buying from seller 1. Therefore, for all values of b^p, reaction functions tend to look more like the case where $b^p = 0$.

Figure B.7: Pricing Best Response Functions in State (30,1) for Different Assumed Continuation Probabilities for Seller 2 (λ_2).

(a) $b^p = 0$



(b) $b^p = 1$



C Algorithm for Establishing Existence of an Accommodative Equilibrium

Definition *An equilibrium is **accommodative** if* $\lambda_1(e_1, e_2) = \lambda_2(e_1, e_2) = 1$ *for all states* (e_1, e_2) *where* $e_1 > 0$ *and* $e_2 > 0$.

In an accommodative equilibrium there is no exit by active sellers. If the industry starts off in state (1,1), it is guaranteed to arrive in state (M, M) in an accommodative equilibrium. This definition is the same as in BDK (2019), Appendix B.

C.1 Existence of an Accommodative Equilibrium

We establish whether an accommodative equilibrium exists by solving, using `fsolve` in MATLAB, for equilibrium prices and values assuming that there is no exit from any duopoly state, and then verifying that it is always optimal for each duopolist to continue in every duopoly state by checking that $\beta V^S(e_1, e_2)$ is greater than the highest possible scrap value.

C.2 Are Accommodative Equilibria Likely to Be Unique?

In an accommodative equilibrium the game is guaranteed to eventually end up in state (M, M), and remain there, and once a state has been left, because one of the sellers has made a sale and increased its know-how, it is guaranteed that the game will not return to it. This feature would guarantee a unique equilibrium if it is the case that there is a unique pricing equilibrium in any state given continuation values if the state changes. However, even though it can be shown that there is a unique price equilibrium in a one-shot Nash pricing game with a multinomial logit demand and an outside good that has a fixed price (e.g., Mizuno (2003)), this result is not sufficient in our model where the prices in the stage game affect sellers' continuation values (and a strategic buyer's continuation value if $b^p > 0$) if no sale is made.[49] The intuition for multiplicity would be that "at a low price equilibrium, each seller has a low opportunity cost of making a sale (when the other seller does not make a sale) as the state is unprofitable, whereas at a high price equilibrium, the opportunity

[49]The result is sufficient for state (M, M) as, whatever the buyer does, the state will be (M, M) in the next period. Therefore, the seller's pricing incentives in a Markov Perfect Equilibrium, will be the same as in a one-shot game.

cost of making a sale is higher". Note that this logic would tend to unravel with a strategic buyer, who would recognize that the possibility of them being the chosen buyer in the next period, which would make them keener to buy from one of the sellers when prices are high, lowering the probability that the state remains the same.

However, in practice, we have not found any examples of states with more than one accommodative pricing equilibrium despite extensive attempts to find an example for different values of b^p. One likely explanation for this is that the assumed value of $v_i = 10$ implies that the probability of the state remaining the same at prices that are close to equilibrium prices is small. For example, for all of the duopoly prices shown in text Table 1 the probability that the outside good is chosen is less than 0.02, and typically less than 0.01.

D Additional Results

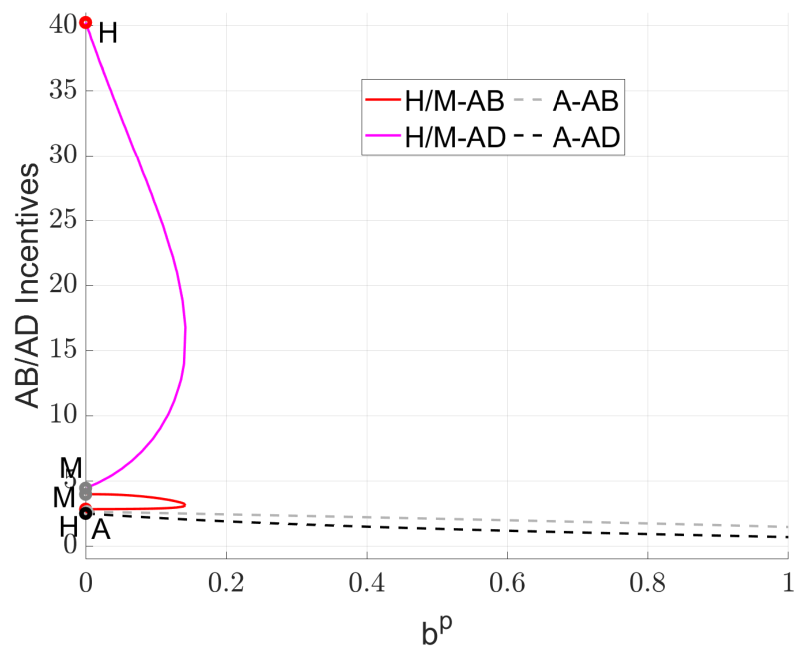
D.1 Equilibrium Buyer and Seller Incentives on b^p-Homotopy Paths for the Illustrative Parameters

Figure D.1(a) shows the equilibrium advantage-building and denying incentives for seller 1 in state (3,1). The decline in seller 1's demand and the falling probability that seller 2 will exit causes seller 1's advantage-denying incentive to fall sharply as we move from the High-HHI baseline equilibrium.

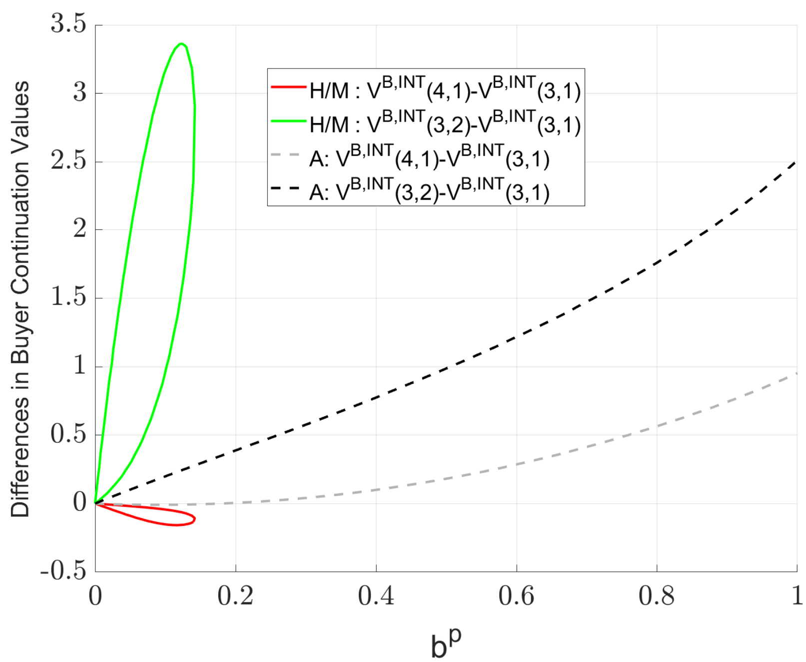
Figure D.1(b) shows the equilibrium dynamic incentives of a strategic buyer in state (3,1), measured by the change in the chosen buyer's continuation values when, compared to not buying, it buys from seller 1 ($V^{B,INT}(4,1) - V^{B,INT}(3,1)$) or seller 2 ($V^{B,INT}(3,2) - V^{B,INT}(3,1)$). These incentives are zero in all of the equilibria when $b^p = 0$. As b^p rises, the dynamic incentive to buy from seller 2 increases sharply in the non-accommodative equilibria, while there is an increasing dynamic disincentive to buy from seller 1. In an accommodative equilibrium there is a positive dynamic incentive to buy from the laggard as this lowers future prices, and, for $b^p > 0.2$ an incentive to buy from the leader which, relative to no purchase, lowers future costs.

Figure D.1: Equilibrium Dynamic Incentives Along b^p-Homotopy Paths for the Illustrative Parameters. H = High-HHI, M = Mid-HHI and A = Accommodative Baseline Equilibria, and AB=Advantage-Building and AD=Advantage-Denying Incentives. Panels (a) and (b) hold seller strategies fixed at baseline equilibrium values.

(a) Seller 1 Equilibrium Incentives



(b) Buyer Equilibrium Incentives



D.2 Additional Welfare Results for the Illustrative Parameters

Text Figure 3 shows that, for $b^p = 0$, the present value of consumer surplus (PV CS) is highest in the Mid-HHI equilibrium and lowest in the High-HHI equilibrium, whereas the present value of total surplus (PV TS) is highest in the accommodative equilibrium and lowest in the High-HHI equilibrium. As b^p increases, both measures of surplus fall in the accommodative equilibrium as prices tend to increase.

The game will be in states (M, M) or $(M, 0)$ in the long-run, so that long-run expected consumer surplus will be higher in the accommodative equilibrium where (M, M) is the certain long-run outcome. PV CS is therefore higher in Mid-HHI equilibrium only because initial prices are lower, while the probability that the industry becomes a monopoly is not too large. To illustrate what happens to welfare in the first part of the game, Figure D.2(a) and (b) show the expected surplus measures for the first ten periods of a game beginning at (1,1). Note that the reported numbers are sums and there is no discounting.

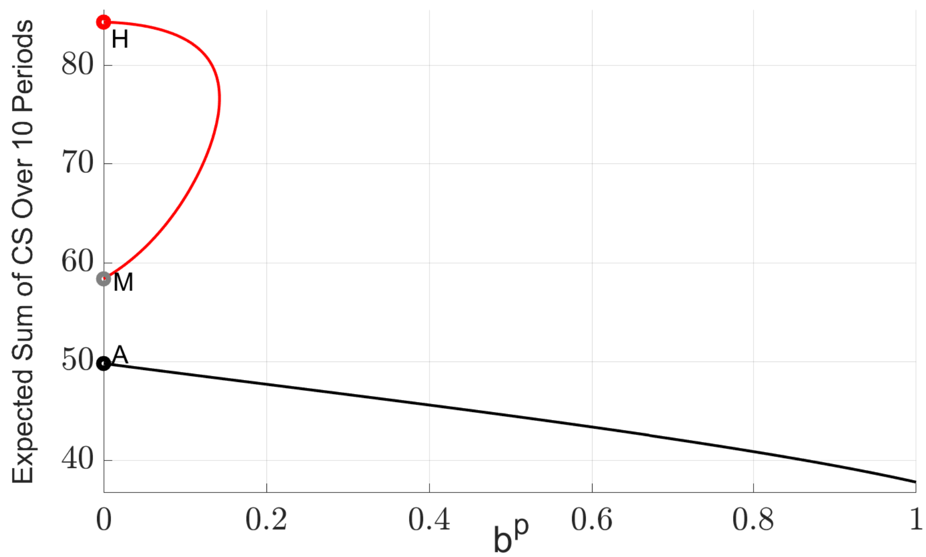
During the first ten periods, consumer surplus is highest in the High-HHI equilibrium due to the very low duopoly prices when one firm has not made a sale. This also tends to increase total surplus. Total surplus is also increased by the reduction in production costs which results from one seller tending to make most of the sales. This is illustrated in Figure D.2(c), which shows the sum of production costs over the first ten periods. The effect that strategic buyer behavior increases prices in the accommodative equilibrium causes both measures of surplus to fall in the accommodative equilibrium as b^p is increased.

The NPV of total surplus is affected by the number of sales that are made and the costs of production. Figure D.3(a) shows that the expected discounted production cost per sale is highest in the accommodative equilibrium, due to slower early learning, and it is lowest in the High-HHI equilibrium, where learning will tend to be quickest.[50] Figure D.3(b) shows the discounted total number of sales that are made. Even though low prices mean that more sales are made at the very beginning of the game in the High-HHI and Mid-HHI equilibria, the discounted number of sales is highest in the accommodative model as, despite higher average production costs, long-run margins are low.

[50] The reported number is the expected discounted total sum of production costs divided by the expected discounted total number of sales.

Figure D.2: Equilibrium Expected Consumer Surplus, Total Surplus and Production Costs Over the First 10 Periods for a Game Starting in State $(1, 1)$ Along b^p-Homotopy Paths for the Illustrative Parameters. The black line traces the homotopy path from the Accommodative (A) baseline equilibrium. The red line traces the overlapping paths from the High-HHI (H) and Mid-HHI (M) baseline equilibria.

(a) Consumer Surplus



(b) Total Surplus

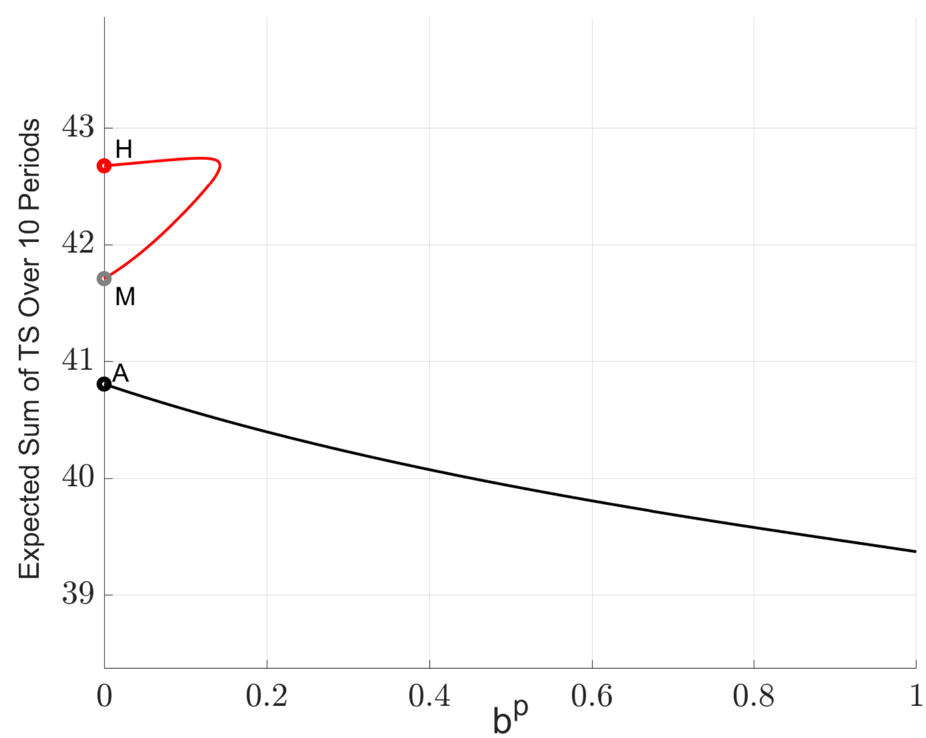


Figure D.2: cont.

(c) Expected Production Costs Over the First 10 Periods of the Game

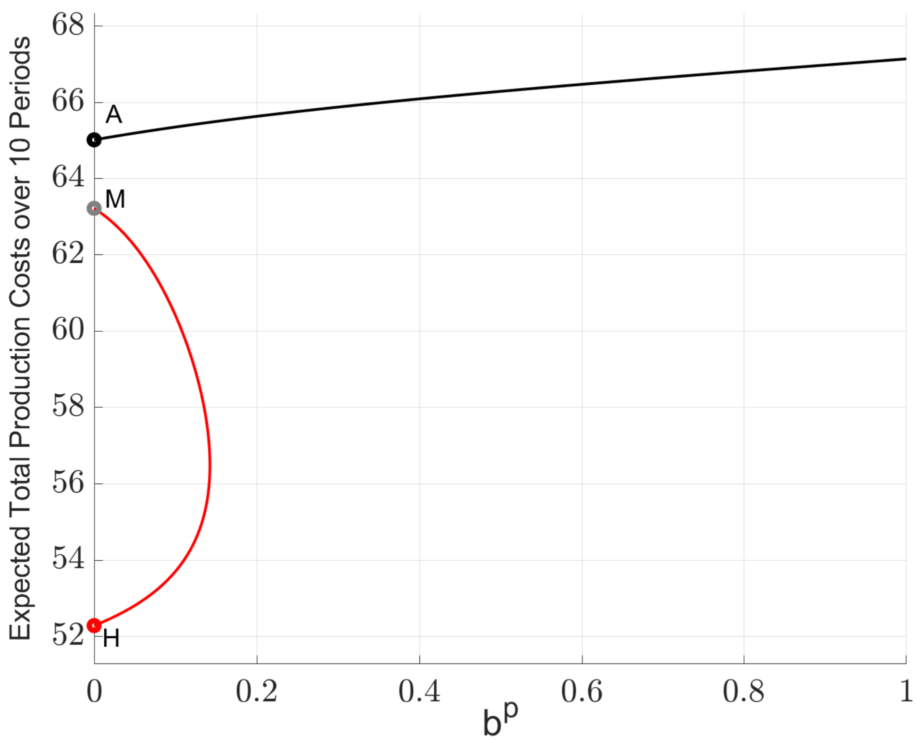
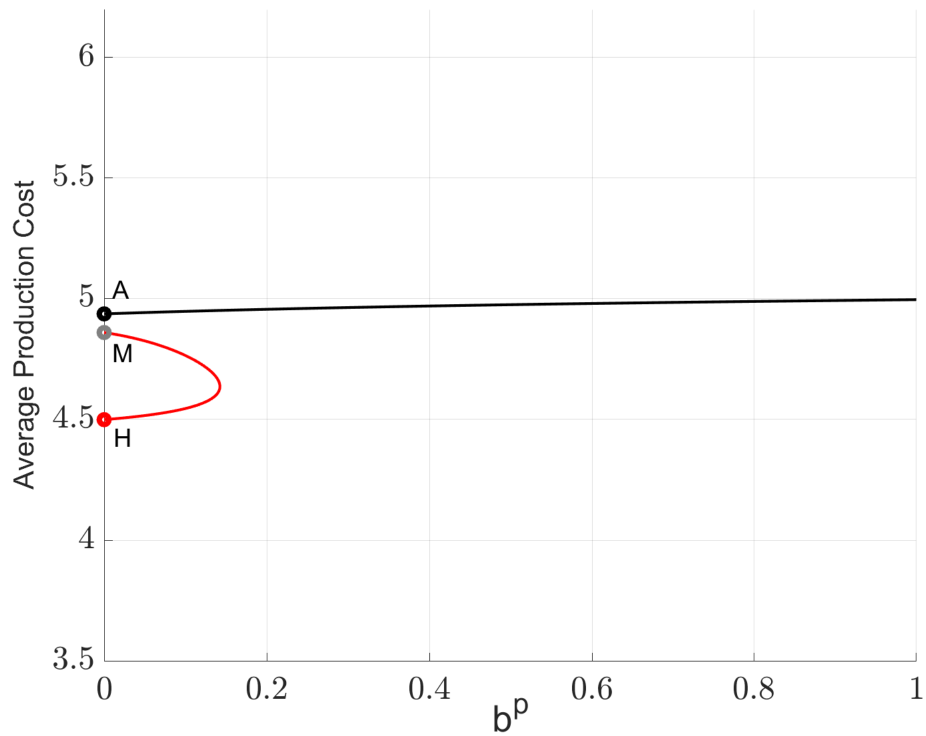
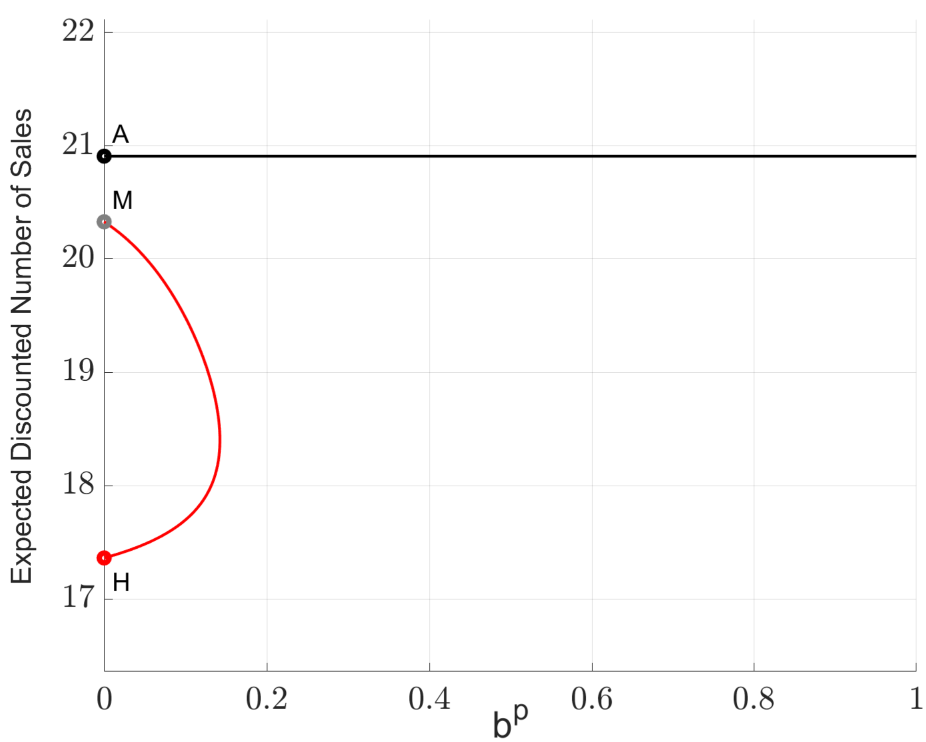


Figure D.3: Expected Present Value of Per-Sale Production Costs and the Expected Present Value (i.e., Discounted) Number of Sales for a Game Starting in State $(1, 1)$ Along b^p-Homotopy Paths for the Illustrative Parameters.

(a) Expected Discounted Per-Sale Production Costs



(b) Expected Discounted Number of Sales

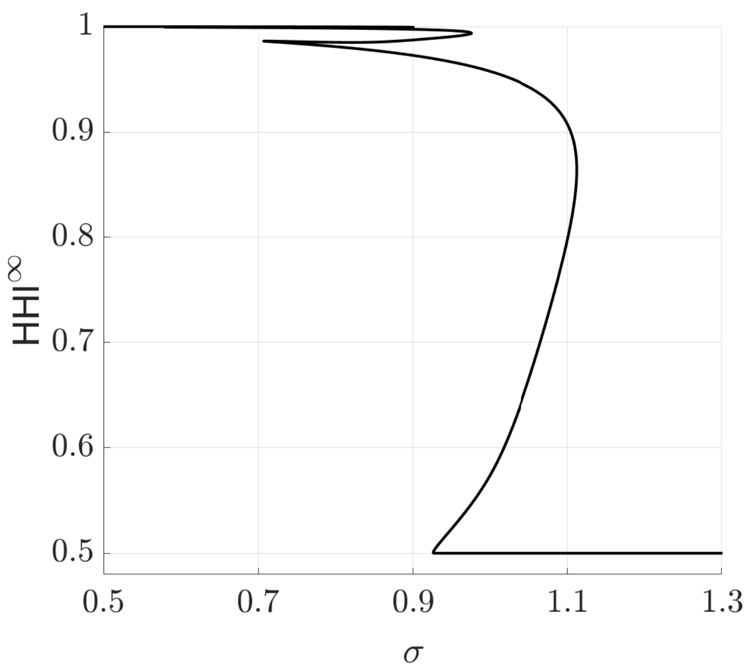


D.3 ρ and σ-Homotopy Paths for $b^p = 0$

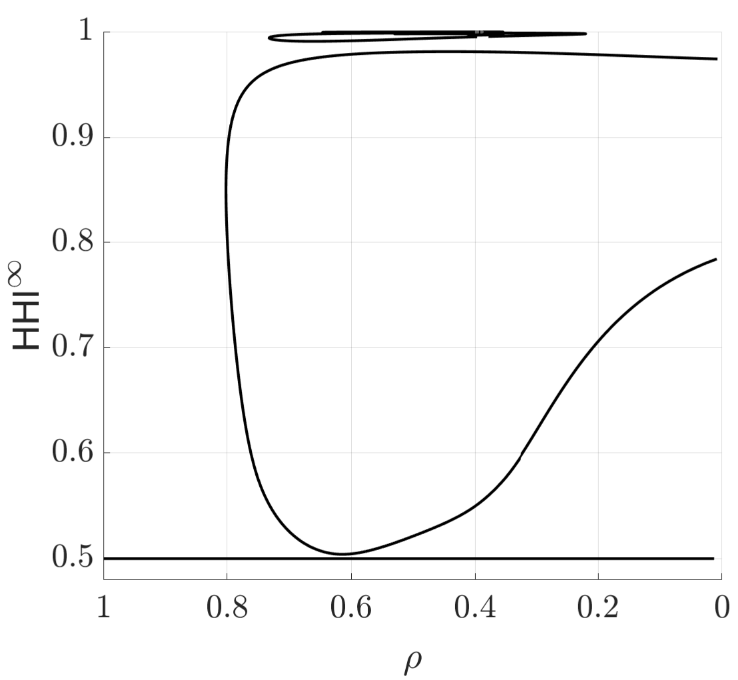
Text Figure 4(a)-(d) show ρ and σ homotopy paths for 11 different values of b^p. We reproduce the HHI^∞ plots for $b^p = 0$ in Figure D.4 for clarity, and so they can be compared with the figures in BDK1, Figure 2, panels A and B.

Figure D.4: Expected Long-Run HHI (HHI^{∞}) for Equilibria Identified by ρ- and σ-Homotopies when $b^p = 0$ and the Other Parameters are at their Illustrative Values.

(a) σ-Homtopies ($\rho = 0.75$)



(b) ρ-Homotopies ($\sigma = 1$)

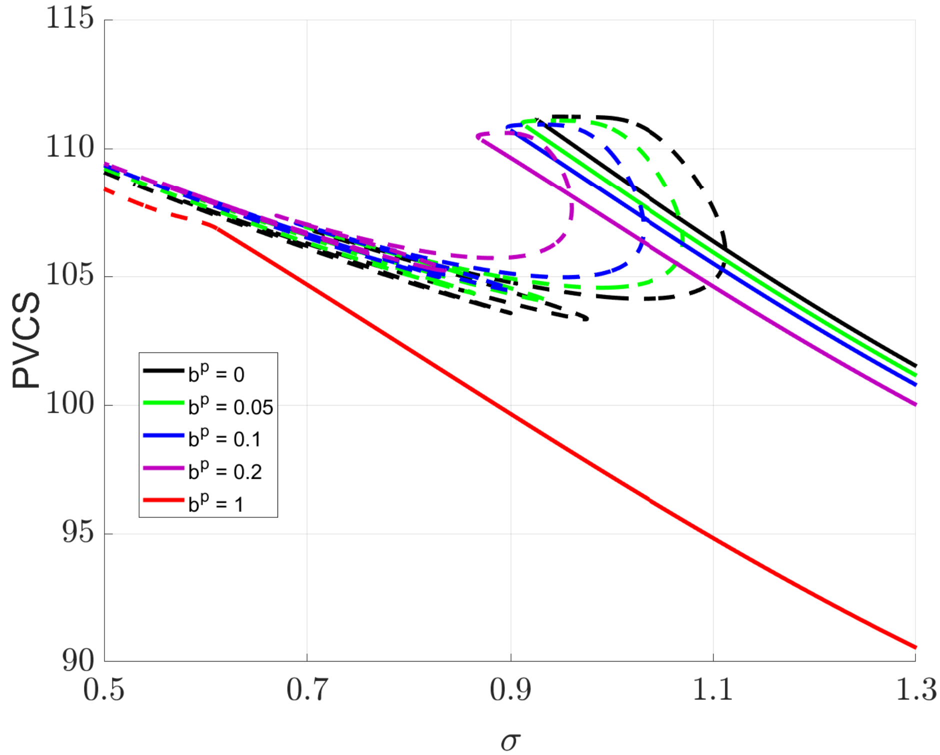


D.4 NPV of Consumer and Total Surplus on σ-Homotopy Paths for Multiple $b^p = 0$.

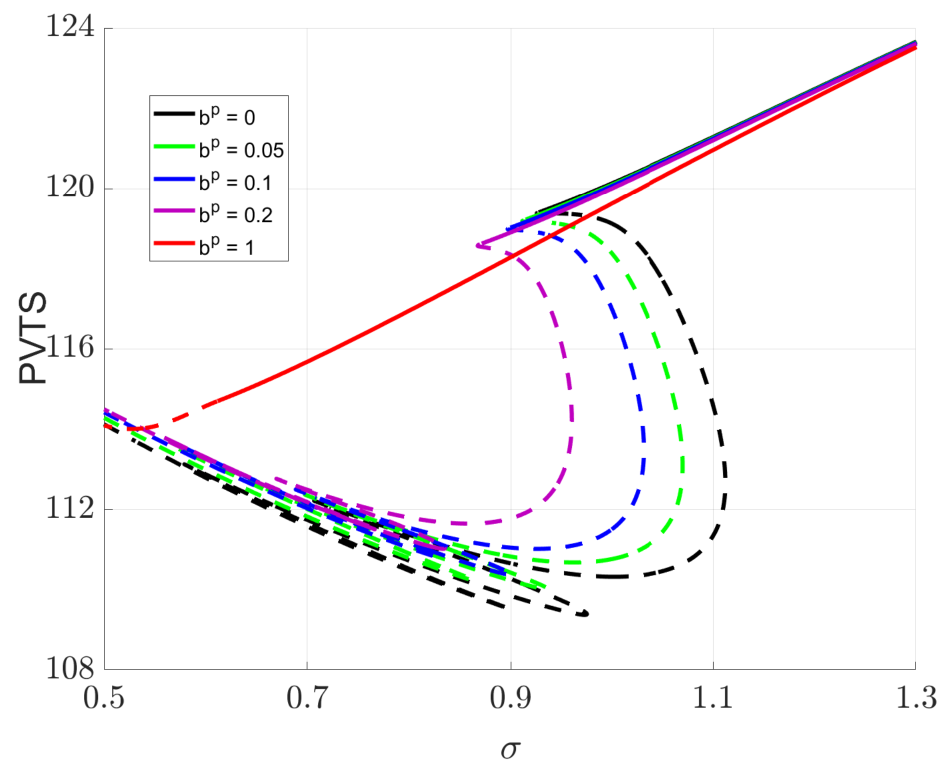
Text Figures 4(e) and (f) show the present value of consumer (PV CS) and total surplus (PV TS) for equilibria on ρ-homotopy paths for 11 different values of b^p. Here we provide similar plots for the σ-homotopies.

Figure D.5: Expected Present Value of Consumer and Total Surplus for Equilibria Along σ-Homotopy Paths for Multiple b^ps with Other Parameters are at their Illustrative Values.

(a) Consumer Surplus



(b) Total Surplus



E Extensions

Section 5 adapts our model in four ways to investigate how small changes to our very stylized assumptions affect our results. In this Appendix we detail these extensions and present some additional results.

E.1 Extension 1: Mixture of Strategic and Non-Strategic Buyers.

In this extension we assume that there are two types of buyers:

1. a mass of atomistic (A) buyers who, if they are chosen to the buyer, assume that they will never be in the market again (i.e., they act as if $b^p = 0$); and,

2. a group of 4 symmetric, strategic (NA, non-atomistic) buyers.

Each period nature picks a strategic buyer with probability γ, in which case each of the four strategic buyers is chosen with equal probability. Otherwise, an atomistic buyer is chosen. The sellers observe the chosen buyer's type before they set prices. If $\gamma = 0$, all buyers are atomistic and equilibrium play corresponds to play in the original BDK model. We run γ-homotopies, for the illustrative parameters, from the three $\gamma = 0$ equilibria.

E.1.1 Equilibrium Equations.

Values of the sellers and the strategic buyers are defined before nature has selected the chosen buyer's type (or the chosen buyer's identity). The values of atomistic buyers are equal to zero, so the only additional set of equations that we have to solve are the pricing first-order conditions of the sellers when selling to atomistic buyers.

<u>Beginning of period value for seller 1 (V_1^S):</u>

$$V_1^S(\mathbf{e}) - (1-\gamma)D_1^A(p^A(\mathbf{e}),\mathbf{e})(p_1^A(\mathbf{e}) - c_1(e_1)) - \gamma D_1^{NA}(p^{NA}(\mathbf{e}),\mathbf{e})(p_1^{NA}(\mathbf{e}) - c_1(e_1)) - \sum_{k=0,1,2} ((1-\gamma)D_k^A(p^A(\mathbf{e}),\mathbf{e}) + \gamma D_k^{NA}(p^{NA}(\mathbf{e}),\mathbf{e}))V_1^{S,INT}(\mathbf{e}'_k) = 0 \tag{E.1}$$

where

$$D_k^A(p^A,\mathbf{e})=\frac{\exp(v_k-p_k^A)}{\sum_{j=0,1,2}\exp(v_j-p_j^A)},\ D_k^{NA}(p^{NA},\mathbf{e})=\frac{\exp(v_k-p_k^{NA}+V^{INT,NA}(\mathbf{e}'_k))}{\sum_{k=0,1,2}\exp(v_j-p_j^{NA}+V^{INT,NA}(\mathbf{e}'_j))},\tag{E.2}$$

$\mathbf{e}'_1=(\min(e_1+1,M),e_2)$, $\mathbf{e}'_2=(e_1,\min(e_2+1,M))$ and $\mathbf{e}'_0=(e_1,e_2)$, i.e., the states that the game will transition to if there is a purchase from seller 1 or seller 2, or no purchase, respectively.

Value for seller 1 before entry/exit stage $(V_1^{S,INT})$:

$$V_1^{S,INT}(\mathbf{e})-\left(\begin{array}{c}\beta\lambda_1(\mathbf{e})\lambda_2(\mathbf{e})V_1^S(\mathbf{e})+\beta\lambda_1(\mathbf{e})(1-\lambda_2(\mathbf{e}))V_1^S(e_1,0)+\\(1-\lambda_1(\mathbf{e}))E(X|\lambda_1(\mathbf{e}))\end{array}\right)=0\tag{E.3}$$

for $\mathbf{e}=(e_1,e_2)$ where $e_1,e_2>0$, with similar equations when one or both sellers is a potential entrant. $E(X|\lambda_1(\mathbf{e}))$ is the expected scrap value when seller 1 chooses to exit with probability $1-\lambda_1(\mathbf{e})$.

First-order condition for seller 1's price to non-strategic buyers (p_1^A) if $e_1>0$:

$$D_1^A(p^A(\mathbf{e}),\mathbf{e})+\sum_{k=0,1,2}\frac{\partial D_k^A(p^A(\mathbf{e}),\mathbf{e})}{\partial p_1^A}V_1^{S,INT}(\mathbf{e}'_k)+\left(p_1^A(\mathbf{e})-c_1(e_1)\right)\frac{\partial D_1^A(p^A(\mathbf{e}),\mathbf{e})}{\partial p_1^A}=0\tag{E.4}$$

First-order condition for seller 1's price to strategic buyers (p_1^{NA}) if $e_1>0$:

$$D_1^{NA}(p^{NA}(\mathbf{e}),\mathbf{e})+\sum_{k=0,1,2}\frac{\partial D_k^{NA}(p^{NA}(\mathbf{e}),\mathbf{e})}{\partial p_1^{NA}}V_1^{S,INT}(\mathbf{e}'_k)+\left(p_1^{NA}(\mathbf{e})-c_1(e_1)\right)\frac{\partial D_1^{NA}(p^{NA}(\mathbf{e}),\mathbf{e})}{\partial p_1^{NA}}=0\tag{E.5}$$

Seller 1's continuation probability in entry/exit stage (λ_1):

$$\lambda_1(\mathbf{e})-F_{\text{enter}}(\beta\left[\lambda_2(\mathbf{e})V_1^S(1,e_2)+(1-\lambda_2(\mathbf{e}))V_1^S(1,0)\right])=0\text{ if }e_1=0\tag{E.6}$$

$$\lambda_1(\mathbf{e})-F_{\text{scrap}}(\beta\left[\lambda_2(\mathbf{e})V_1^S(e_1,\max(1,e_2))+(1-\lambda_2(\mathbf{e}))V_1^S(e_1,0)\right])=0\text{ if }e_1>0\tag{E.7}$$

Value for strategic buyer before entry/exit stage $(V^{INT,NA})$:

$$V^{INT,NA}(\mathbf{e}) - \beta \left(\sum_{\mathbf{e}'} \Pr(\mathbf{e}'|\mathbf{e}, \lambda_1(\mathbf{e}), \lambda_2(\mathbf{e})) V^{NA}(\mathbf{e}') \right) = 0. \tag{E.8}$$

where the sum is over the states that the game may transition to given entry/exit choices. Seller symmetry implies that, for buyers, $V^{INT,NA}(e_1, e_2) = V^{INT,NA}(e_2, e_1)$ and $V^{NA}(e_1, e_2) = V^{NA}(e_2, e_1)$.

Beginning of period strategic buyer value (V^{NA}) :

$$V^{NA}(\mathbf{e}) - \frac{1}{4}\gamma \log \left(\sum_{k=0,1,2} \exp\left(v_k - p_k^{NA} + V^{INT,NA}(\mathbf{e}'_k)\right) \right) - (1-\gamma) \sum_{k=0,1,2} D_k^A(p^A(\mathbf{e}), \mathbf{e}) V^{INT,NA}(\mathbf{e}'_k) - \tag{E.9}$$
$$\gamma(1 - \frac{1}{4}) \sum_{k=0,1,2} D_k^{NA}(p^{NA}(\mathbf{e}), \mathbf{e}) V^{INT,NA}(\mathbf{e}'_k) = 0$$

where $\frac{1}{4}$ is the probability that a given strategic buyer is chosen when one of them is selected.

E.2 Extension 2: Buyers with Persistent Preferences Over Sellers.

The Section 2 model also assumes that buyers always have identical preferences over sellers up to iid preference shocks. In reality, buyers may have systematic preferences for a particular seller (for example, because of geographic location or greater compatibility with existing equipment). We therefore extend the Section 2 model by assuming that there are equal numbers of two types of buyers. Type 1's indirect utility when it purchases from sellers 1 and 2 respectively are $v_1 + \frac{\theta}{2} - p_1 + \epsilon_1$ and $v_2 - \frac{\theta}{2} - p_2 + \epsilon_2$ respectively. For type 2 buyers, the signs on the $\frac{\theta}{2}$ terms are reversed. Sellers recognize the type of the buyer before setting prices. The model is equivalent to the Section 2 model when $\theta = 0$. Intuitively, it will become more attractive for a seller to remain in the market as θ increases, even when it has a marginal cost disadvantage, as it will have an increasing advantage when selling to half of the market.

E.2.1 Equilibrium Equations.

To the equations of the Section 2 model are added type-specific first-order conditions for prices, and equations for the values and intermediate values.[51] For example,

First-order condition for seller 1's price (p_1^{type1}) if $e_1 > 0$:

$$D_1^{type1}(p^{type1}(\mathbf{e}), \mathbf{e}) + \sum_{k=0,1,2} \frac{\partial D_k^{type1}(p(\mathbf{e}), \mathbf{e})}{\partial p_1^{type1}} V_1^{S,INT}(\mathbf{e}'_k) + \left(p_1^{type1}(\mathbf{e}) - c_1(e_1)\right) \frac{\partial D_1^{type1}(p^{type1}(\mathbf{e}), \mathbf{e})}{\partial p_1^{type1}} = 0 \tag{E.10}$$

Value for type 1 buyer before entry/exit stage ($V^{type1,INT}$) :

$$V^{type1,INT}(\mathbf{e}) - \beta \left(\sum_{\mathbf{e}'} \Pr(\mathbf{e}'|\mathbf{e}, \lambda_1(\mathbf{e}), \lambda_2(\mathbf{e})) V^{type1}(\mathbf{e}') \right) = 0. \tag{E.11}$$

Type 1 buyer value (V^{type1}) :

$$V^{type1}(\mathbf{e}) - b^p \log \left(\sum_{k=0,1,2} \exp \left(v_k + [I(k=1) - I(k=2)] \frac{\theta}{2} - p_k^{type1} + V^{type1,INT}(\mathbf{e}'_k) \right) \right) - \tag{E.12}$$

$$(\frac{1}{2} - b^p) \sum_{k=0,1,2} D_k^{type1}(p^{type1}(\mathbf{e}), \mathbf{e}) V^{type1,INT}(\mathbf{e}'_k) - \frac{1}{2} \sum_{k=0,1,2} D_k^{type2}(p^{type2}(\mathbf{e}), \mathbf{e}) V^{type1,INT}(\mathbf{e}'_k) = 0$$

with similar equations for type 2 buyers. Note that b^p is equal to the unconditional probability that the buyer will be the buyer in a future period, so the value of b^p with a single, rational buyer of each type would be $b^p = 0.5$.

Values for sellers then come from adding across the two types of buyers.

[51] The assumed functional forms imply that there will be symmetry across types, e.g., the price set by seller 1 to a type 1 buyer in state (4,1) will be the same as the price set by seller 2 to a type 2 buyer in state (1,4).

Beginning of period value for seller 1 (V_1^S):

$$V_1^S(\mathbf{e}) - \frac{1}{2}D_1^{type1}(p^{type1}(\mathbf{e}),\mathbf{e})(p^{type1}(\mathbf{e}) - c_1(e_1)) - \frac{1}{2}\sum_{k=0,1,2} D_k^{type1}(p^{type1}(\mathbf{e}),\mathbf{e})V_1^{S,INT}(\mathbf{e}'_k) -$$
$$\frac{1}{2}D_1^{type2}(p^{type2}(\mathbf{e}),\mathbf{e})(p^{type2}(\mathbf{e}) - c_1(e_1)) - \frac{1}{2}\sum_{k=0,1,2} D_k^{type2}(p^{type2}(\mathbf{e}),\mathbf{e})V_1^{S,INT}(\mathbf{e}'_k) = 0 \tag{E.13}$$

where

$$D_i^{type1}(p,\mathbf{e}) = \frac{\exp(v_i + [I(i=1) - I(i=2)]\frac{\theta}{2} - p_i^{type1} + V^{type1,INT}(\mathbf{e}'_i))}{\sum_{k=0,1,2}\exp(v_k + [I(k=1) - I(k=2)]\frac{\theta}{2} - p_k^{type1} + V^{type1,INT}(\mathbf{e}'_k))}. \tag{E.14}$$

$\mathbf{e}'_1 = (\min(e_1+1, M), e_2)$, $\mathbf{e}'_2 = (e_1, \min(e_2+1, M))$ and $\mathbf{e}'_0 = (e_1, e_2)$, i.e., the states that the game will transition to if there is a purchase from seller 1 or seller 2, or no purchase, respectively.

E.3 Extension 3: Bargaining as a Constraint on Monopoly Power.

We consider a permutation of the model where we assume that, in the event that the industry becomes a monopoly, the buyer and seller engage in Nash bargaining rather than the seller simply setting a price. This formulation is somewhat ad-hoc because the Nash bargaining approach assumes that the buyer and seller have complete information about their values (i.e., the buyer's ϵs are publicly observed) whereas, to keep the model as similar to the BDK model as possible, we maintain the assumption that a buyer's ϵs are private information in duopoly states. However, the advantage of the Nash bargaining formulation is that it allows us to vary a single parameter, τ, that measures the buyer's share of the surplus from trade in monopoly states.

E.3.1 Details.

The equations for states with two active sellers are the same as for the Section 2 model. The following are the equations for a monopoly state $\mathbf{e} = (e_1 < M, 0)$.

Probability of trade when seller 1 is the monopolist

$$D_1 = \frac{\exp(v_1 + V^{S,INT}(\mathbf{e}_1') + V^{B,INT}(\mathbf{e}_1') - c(e_1))}{\left(\begin{array}{c} \exp(v_1 + V^{S,INT}(\mathbf{e}_1') + V^{B,INT}(\mathbf{e}_1') - c(e_1)) + \\ \exp(V^{S,INT}(\mathbf{e}) + V^{B,INT}(\mathbf{e})) \end{array}\right)} \tag{E.15}$$

Beginning of period value for seller 1 (V_1^S):

$$V_1^S(\mathbf{e}) - D_1(\mathbf{e})(p(\mathbf{e}) - c_1(e_1)) - D_1(\mathbf{e})V_1^{S,INT}(\mathbf{e}_1') - (1 - D_1(\mathbf{e}))V_1^{S,INT}(\mathbf{e}) = 0 \tag{E.16}$$

Intermediate value for seller 1 ($V_1^{S,INT}$) :

$$V_1^{S,INT}(\mathbf{e}) - \left(\begin{array}{c} \beta\lambda_1(\mathbf{e})\lambda_2(\mathbf{e})V_1^S(e_1, 1) + \beta\lambda_1(\mathbf{e})(1 - \lambda_2(\mathbf{e}))V_1^S(\mathbf{e}) + \\ (1 - \lambda_1(\mathbf{e}))E(X|\lambda_1(\mathbf{e})) \end{array}\right) = 0 \tag{E.17}$$

Value for buyer before entry/exit stage ($V^{B,INT}$) :

$$V^{B,INT}(\mathbf{e}) - \beta\left(\lambda_1(\mathbf{e})\lambda_2(\mathbf{e})V^B(e_1, 1) + \lambda_1(\mathbf{e})(1 - \lambda_2(\mathbf{e}))V^B(\mathbf{e})\right) = 0 \tag{E.18}$$

Beginning of period buyer value (V^B) :

$$\begin{aligned} &V^B(\mathbf{e}) - b^p\left(D_1(\mathbf{e})(v_1 - p(\mathbf{e}, \tau) - \log(D_1) + V^{B,INT}(\mathbf{e}_1')) + (1 - D_1(\mathbf{e}))(-\log(1 - D_1) + V^{B,INT}(\mathbf{e}))\right) \\ &-(1 - b^p)\left(D_1(\mathbf{e})V^{B,INT}(\mathbf{e}_1') + (1 - D_1(\mathbf{e}))V^{B,INT}(\mathbf{e})\right) = 0 \end{aligned} \tag{E.19}$$

Expected price in the event of trade is

$$p(\mathbf{e}) = \tau(c(e_1) + V^{S,INT}(\mathbf{e}) - V^{S,INT}(\mathbf{e}_1')) + (1-\tau)(\underbrace{v_1 - \log(D_1)}_{\text{exp. value of v+}\varepsilon_1\text{ given trade}} + V^{B,INT}(\mathbf{e}_1') - \tag{E.20}$$

$$\underbrace{\frac{(1-D_1)\log(1-D_1)}{D_1}}_{\text{exp. value of }\varepsilon_0\text{when trade occurs}} - V^{B,INT}(\mathbf{e})) \text{ for } \mathbf{e} = (e_i, 0) \text{ and } \mathbf{e}_1' = (e_1 + 1, 0)$$

E.4 Extension 4: Buyer Discount Factors.

We investigate whether variation in b^p has a similar effect to variation in buyer patience using a model where $b^p = 1$ (i.e., monopsony) but the buyer's discount factor, $\beta^B \leq \beta = \frac{1}{1.05}$, the assumed discount factor of the sellers. The equations are the same as for the Section 2 model except that β in the $V^{B,INT}$ equation is replaced by β^B.

E.4.1 Effect of Variation on β^B on Seller 2 Demand Given Baseline Equilibrium Seller Strategies.

Figure E.1 shows the demand curve for seller 2 in state (3,1) when we assume that sellers use their baseline equilibrium seller strategies in all states, but we assume that there is a single strategic buyer ($b^p = 1$) with different discount factors. When sellers use accommodative equilibrium strategies, an increase in buyer patience tends to move demand towards seller 2 (the laggard), in the same way that an increase in b^p moved demand towards seller 2 in text Figure 2(a). However, in the Mid- and High-HHI equilibria, increases in β^B actually shift demand away from seller 2 until $\beta^B > 0.5$ in the High-HHI case, and until $\beta^B > 0.7$ in the Mid-HHI case, reflecting the fact that in these equilibria prices in state (4,1) are lower than in state (3,2) and that the loss that the buyer will experience from monopoly is likely to occur further into the future.

Figure E.1: Seller 2 Demand in State (3,1) as a Function of β^B.