



Discussion of “On the possibility of krusell-smith equilibria”

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ABSTRACT

Broer et al. (2021) provide an “information criterion” and show the possibility of non-existence of the well-studied equilibrium of a workhorse heterogeneous agent model with aggregate uncertainty and incomplete markets, as exemplified in Krusell and Smith (1998) and as routinely solved thereafter. This discussion highly advocates the first-order importance of checking this information criterion against the commonly assumed equilibrium definition for a class of heterogeneous agent models. Though, I note that while the endogenous learning is critical, the proposed information criterion may be extended to capture the optimality of information structures across agents. A few other extensions are discussed to delve deeper into the conclusions and the general implications of this paper.

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Broer et al. (2021) reconsider the key assumption of agents having symmetric, common and near perfect information as routinely built in the equilibrium definition and in the classic methodology of solving a heterogeneous agent model with incomplete markets and aggregate risk (Krusell and Smith, 1998; thereafter, the KS Model). The paper in particular examines the existence of such equilibrium by relaxing this assumption. It proposes an “information criterion” and demonstrates that the well-studied and routinely defined equilibrium may not exist.

By allowing optimizing agents to endogenize the learning decisions and rationally differ in their information sets ex-post in an otherwise standard KS model, Broer et al. (2021) find very interesting and striking results. First, agents optimally choose whether or not to learn about the aggregate state variables given some information acquisition cost. It follows that learning decisions across agents can be driven by the wealth distribution. Second, per the presence of strategic substitutability of saving decisions, drawing information among learning agents discourages extra information acquisition of others. These results suggest that the typical equilibrium assuming common and close to perfect information across all agents as commonly solved in this class of models may not even exist, even if learning can be costly but reasonably cheap.

The general takeaway from the paper is very big, about which I am very glad. That is, if the information acquisition decision is part of the agents’ optimization game, there is no way that all agents are bounded rational. It means that not all agents would acquire information in the “correct” equilibrium once adjusted for the information endogeneity. Given that not all agents would ever learn about the aggregate TFP and the first moment of the distribution of idiosyncratic endogenous state variable, i.e. capital stock in the KS model, a representative-agent type of equilibrium is not going to approximate well the dynamics of a model of heterogeneous agents when subject to information heterogeneity and endogeneity. More ambitiously, we might follow this line of reasoning and argue that all previous work following the KS alike approaches that solve and study the economies of heterogeneous agents based on an approximated equilibrium may be simply erroneous. The authors do have very good arguments about this point because there is very strong evidence documenting distribution of heterogeneous beliefs and its distributional variations (Coibion et al., 2018; 2021).

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In general, the paper of [Broer et al. \(2021\)](#) is very well executed and cleanly polished. With both analytical and numerical results, the insights of the paper are clear and are expected to generate broader impacts. In the following, I am listing a few things to delve deeper into the paper conclusions and possibly push a bit further for future research endeavors.

Firstly, the modeling choice of endogenous learning is silent about the optimality of the information structure on which learning or not is an option for agents. That is, it is unspecified or undisciplined regarding what is the exact set of variables that all agents or a fraction of agents should draw information about. In the KS baseline model, agents observe the current aggregate TFP and the aggregate capital stock, which are sufficient to well forecast the future aggregate capital, wage and rate of return to capital without knowing the exact distribution of agents and its evolution over time. That means, households do not err too much by knowing about just the aggregates and this generically imposed perceived law of motion of agents turns out to capture super well the true distributional transition of idiosyncratic and aggregate states over time. This is exactly the reason why the verified (or, numerically converged) perceived law of motion for aggregate capital in the KS model has a R2 of 99.99% in its regression setting. Apparently, while leaving the question why the TFP and aggregate capital matter that much widely open, Krusell and Smith and a line of following macroeconomists were not attracted to decipher further on this. That is, why aggregate states of the first moments are key to the optimal set of information for every agent in the heterogeneous agent model is unclear ever since. [Broer et al. \(2021\)](#) are not trying to provide answer to this question either, which would be otherwise a tremendous contribution in this line of literature. Rather, in the simple 2-period model of [Broer et al. \(2021\)](#), the authors then impose the learning objects for all agents, that is, wage and the return to capital. Also in the full-fledged model, the authors are doing experiments with various degrees of information truncation, i.e. iteratively dropping TFP and the aggregate capital as variables of learning interests. As a result, [Broer et al. \(2021\)](#) are able to show learning is important only to the perspective of some agents who are rich enough to care about wage income and who are poor enough that learning triggers a marginal gain. However, since KS and all the way till [Broer et al. \(2021\)](#), what are the optimal information set on average and what is the optimal information set for each individual are largely underexplored in heterogeneous agent models. But clearly, there is a line of pecking order about what set of variables should matter the most for some agents that triggers learning. [Broer et al. \(2021\)](#) highlight the importance of learning conditional on imposed information structures while again discussion of the optimality of information structure across agents is missing. This is the key point that may be improved further in this dimension.

My thinking is that the literature on Rational (In)attention may be readily helpful at least to some extent ([Sims, 2003](#)). For example, [Kacperczyk et al. \(2016\)](#) have mutual funds subject to attention constraint face trade-off decisions of learning about aggregate variables vs. idiosyncratic variables when it comes to investing in different risky assets. That is, learning about aggregate variables is not always optimal over time. Another enriched example in this literature emphasizes that investors differ in the information processing capacity for allocating attention to different assets. [Kacperczyk et al. \(2019\)](#) introduces the asymmetry of learning across agents such that some investors are learning about asset prices and some others are optimizing the information structure to better learn about asset payoffs. These are the examples that show both theoretically and empirically the optimality of learning is conditional on attention optimization and the optimality of information sets. Therefore, not only learning is not uniform but also what to learn is not symmetric across agents, which is exactly dependent on the wealth distribution and possibly linked to the information processing capacity. In other words, suppose we are having a richer model with taxes, government's transfers and subsidies, it is not a question that the hand-to-mouth households would not care about wage and investment returns too much like middle-income households would do. However, the hand-to-mouth households for sure still learn and they do care about reservation wage, unemployment insurance and the minimum wage. Similarly, top-percentile households would pay attention to tax evasions and care more about the capital and property tax. In sum, combining both [Broer et al. \(2021\)](#) and attention optimization, it can be a much better model with heterogeneous agents such that in the true equilibrium, not all agents learn about the shared set of variables of interest but agents have differed attention devotion and thus varied information sets up to optimum.

Therefore, there is an issue of attention allocation across variables of importance missing from the current setting. It is not simply about learning or not but it is about learning or not learning about something conditional on its importance. Incorporating this insight of attention allocation in disciplining the information criterion across heterogeneous agent models for future extensions would be extremely helpful. Though, I admit this adds tremendous complexity especially when it comes to dynamic allocation of attention across multivariate variables ([Che and Mierendorff, 2019](#); [Miao et al., 2021](#)) while combining the frontier heterogeneous agent models are already much harder to solve relative to KS baselines ([Kaplan et al., 2018](#)). All these challenges (not simply computational) arise because information decision is not static and its naturally feeding into dynamic loops interacting with state variables that affect agents attention devotion over time.

Secondly, when motivating or elaborating the contribution of the model by referencing to the data of belief updating and heterogeneous expectations, some behavioral perspective may be still useful to discuss for future extensions. For example, the Level-K reasoning, which may exist at least for some time periods among some particular groups of household agents ([Nagel, 1995](#)). In [Broer et al. \(2021\)](#), it is tractable to impose that the fixed set of key variable of interest for learning decisions across agents and then compute and keep track of the higher order beliefs. Alternatively, there is the hierarchy that some agents' state variables of interests are limited within finite orders because of Level-K reasoning when forecasting the others' forecasts. This creates additional information asymmetry across agents that is not due to wealth distribution or attention allocation. I would suggest that adding truncated higher order beliefs for some agents with or without conditional on wealth distribution and tease out the impacts of truncated order of learning and reasoning.

Thirdly, taking as given that the symmetric, near perfect and common information-consistent equilibrium may not exist in general in this class of models, what would be a better information-criterion adjusted approach to solve a model of heterogeneous agents and to better understand macro and micro-level dynamics? Broer et al. (2021) clearly help us a lot to think about the limitations in the existing model solutions and in the interpretations of equilibrium outcomes. In addition, even if the equilibrium solution is corrected and the information heterogeneity along with other asymmetries across agents are all well captured, do macroeconomists know better about aggregate dynamics like growth patterns or business cycles quantitatively or qualitatively? In other words, some insights from the current routine solutions of the equilibrium of heterogeneous agent models would be: higher order moments of the state distributions do not matter that much as expected when fitting the aggregate law of motion in presence of additional non-linearity (Bloom, 2009; Bloom et al., 2018) and significant micro-level lumpy frictions may still aggregate up to smooth time series (Khan and Thomas, 2008; 2013). Do these interpretations change once the information adjustment is applied? It would be very interesting for the future research to propose improved solution methodology but this is only the first step. How do we separate the distributional effects of aggregate shocks and idiosyncratic shock driven by the information disparities from those arising from the rest of the equilibrium conditions? This is the most important step thereafter. It would be also interesting to look at what fractions of aggregate dynamics are driven by learning inequalities, by attention allocations and by real heterogeneities and how these aggregate dynamics would feed back to shape the dynamics of the heterogeneities across agents over time.

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