Bargaining and Dynamic Competition

Shanglyu Deng
University of Macau

Dun Jia
Peking University

Mario Leccese
University of Maryland

Andrew Sweeting*
University of Maryland, CEPR and NBER

April 2024

Abstract

Industries with significant scale economies or learning-by-doing may come to be dominated by a single firm. Economists have studied how likely this is to happen, and whether it is efficient, using models where buyers are price or quantity takers, even though these industries are often also characterized by buyer-seller negotiations. We extend the dynamic "learning-by-doing and forgetting" model of Besanko, Doraszelski, Kryukov, and Satterthwaite (2010) to allow for Nash-in-Nash bargaining over prices. Price-taking and the social planner solution are captured as special cases. We show that sellers' dynamic incentives, market concentration and welfare can change sharply, and non-monotonically, as one moves away from the price-taking assumption. We study the implications of buyer bargaining power for the existence of multiple equilibria, the design of subsidy policies and the welfare effects of policies designed to increase competition.

Keywords: dynamic competition, learning-by-doing, bargaining power, buyer power, multiple equilibria.

JEL codes: C73, D21, D43, L13, L41.

*Contact author: atsweet@umd.edu. Authors ordered alphabetically. The research was supported by a BSOS DRI award from the University of Maryland. We are very grateful to conference and seminar participants at Yale, UC Berkeley, the 2023 International Industrial Organization, CEPR/JIE Applied IO and Northwestern Antitrust conferences. David Besanko, Chris Conlon and John Thanassoulis have provided insightful and helpful discussions. Steve Berry, Jim Dana, Uli Dorazelski, Paul Greico, Steve Kryukov, Carl Shapiro and Dan Vincent have made useful comments on this paper and/or closely related work. We have also benefited from the work of Shen Hui and Xinlu Yao on related projects.

1 Introduction

Industries where some feature of demand or supply may allow a firm that achieves an initial advantage to attain a position of lasting market dominance often attract the attention of policy-makers. For example, industrial and trade policies have been implemented or proposed for industries making electric vehicle batteries, semiconductors, solar panels and aircraft, where scale economies or learning-by-doing (LBD) are important, while, in a different type of setting, antitrust agencies and regulators are increasingly active when addressing the largest players in digital markets characterized by network effects. Even when non-economic concerns motivate policies, models of dynamic competition are required to evaluate their effects.

Existing theoretical or empirical models of dynamic competition in industrial organization (for example, Fudenberg and Tirole (1983), Cabral and Riordan (1994), Benkard (2000), Besanko, Doraszelski, Kryukov, and Satterthwaite (2010), Besanko, Doraszelski, and Kryukov (2014)) and international trade (Dasgupta and Stiglitz (1988), Leahy and Neary (1999), Neary and Leahy (2000)) assume that sellers unilaterally set prices or quantities. These models therefore miss how prices are negotiated in many industries, especially for capital goods, where dynamic competition is important.

This paper examines how bargaining affects outcomes and optimal policies in a model of dynamic price competition, where buyers arrive repeatedly and market structure is endogenous. To study novel interactions between bargaining and dynamics, we extend the well-known dynamic duopoly seller "learning-by-doing (LBD) and forgetting" computational model of Besanko, Doraszelski, Kryukov, and Satterthwaite (2010) (BDKS). Specifically, we replace the assumption that sellers simultaneously set take-it-or-leave it prices each period with a more general assumption that prices are determined by a form of "Nash-in-Nash" bargaining between the buyer and the sellers (Horn and Wolinsky (1988), Collard-Wexler, Gowrisankaran, and Lee (2019)). The Nash-in-Nash structure, embedded in static models, has become widely used to understand business-to-business transactions in healthcare (e.g., Gowrisankaran, Nevo, and Town (2015), Ho and Lee (2017), Grennan (2013)), cable television (e.g., Crawford, Lee, Whinston, and Yurukoglu (2018))) and consumer packaged goods settings (e.g., Draganska, Klapper, and Villas-Boas (2010)).

Assuming that there is a myopic single-unit buyer every period, an assumption that we maintain, and that sellers set prices, BDKS compute symmetric Markov Perfect Nash equilibria, assuming thirty levels of know-how ($M = 30$) for each firm. Equilibrium prices equal a markup plus the seller's opportunity cost of sale. The opportunity cost equals the current production cost, which falls with know-how, less how much the seller's expected future profit increases when it makes a sale (dynamic incentives). BDKS search for equilibria using numerical homotopies and find that equilibria where sellers set low prices in states where they are symmetric, even when they have accumulated significant know-how, and one seller will typically tend to have a significant lead in equilibrium, often co-exist with equilibria where prices are less sensitive to know-how and more symmetric market structures will tend to emerge.

We focus on how the allocation of bargaining power, determined by a single parameter, affects market structure and welfare. Shifting bargaining power to buyers will lower seller markups and, holding the evolution of market structure fixed, will tend to reduce sellers' dynamic incentives. However, markup shrinkage will also lead to the seller with the larger markup, typically the leader, making more sales so that leads last longer. When sellers' bargaining power is not too limited and leads are initially short-lived, this lead lengthening can increase the incentives of a seller to attain and to preserve a lead. We show, for a wide range of empirically relevant learning parameters, that this effect can cause market structure and welfare outcomes to change significantly and quickly as we move away from the standard assumption that sellers make take-it-or-leave-it offers.

We explore how the allocation of bargaining power changes the design and the effects of stylized policies. We show, for example, that a subsidy scheme that would implement the social planner outcome with price-setting can be worse than no scheme at all when sellers only have much greater bargaining power than buyers. We also consider policies that have been considered as likely to promote more symmetric market structures, possibly at the cost of softening competition. We find that the effects of these policies on concentration and welfare can also be sensitive to the assumed allocation of bargaining power.

We also make methodological contributions. We reformulate the equations defining symmetric equilibria in terms of the probability that a laggard makes a sale, rather than prices and values. This reduces the number of equations from $2M^2$ to $\frac{M(M-1)}{2}$, allowing us to provide some limited analytical results for the $M = 2$ model, to use a non-homotopy method to enumerate equilibria when $M = 3$ (allowing us to confirm homotopy results), and to formulate easy-to-solve equations for optimal subsidies even when M is large. We find that, for both $M = 3$ and $M = 30$, the multiplicity of equilibria that is common with price-setting disappears when buyers have more than limited bargaining power.

By extending a price-setting model to allow for bargaining, our paper contributes directly to the literature studying dynamic competition with LBD that spans the analytic insights of Cabral and Riordan (1994), the computational analyses of BDKS, Besanko, Doraszelski, and Kryukov (2014), Besanko, Doraszelski, and Kryukov (2019a), Besanko, Doraszelski, and Kryukov (2019b) and Sweeting, Jia, Hui, and Yao (2022) (SJHY), and the empirical analyses of mergers and industrial policies by Benkard (2004), Kim (2014), Kalouptsidi (2018), An and Zhao (2019), and Barwick, Kalouptsidi, and Zahur (2019).[1] Our policy examples are very stylized, but they suggest how allowing for bargaining could also change the policy recommendations from richer models. SJHY assume price-setting but extend the Besanko, Doraszelski, and Kryukov (2014) model to allow for forward-looking buyers who expect to purchase in

[1]A distinct literature on industry dynamics (Abbring and Campbell (2010) and Abbring, Campbell, Tilly, and Yang (2018)) assumes that firms are symmetric to guarantee equilibrium uniqueness in order to simplfiy an analysis of how industries respond to aggregate shocks.

some share of future periods. We will maintain BDKS's assumption that buyers are myopic in this paper, except when we solve the social planner's problem by assuming that there is a long-lived buyer who captures all surplus.

The Besanko, Doraszelski and Kryukov papers use a model ("BDK model") where sellers can enter or exit, and there is no forgetting. We focus on the BDKS model for three reasons. First, reallocating bargaining power away from sellers in a model with fixed costs (or entry costs and scrap values) could cause the industry to disappear entirely. This effect is no doubt important, but it is distinct from the effects of bargaining power on dynamic incentives and leadership on which we focus. Second, our reformulation has particular benefits in the BDKS model where there is no outside good, although we allow for an outside good as a robustness check. Third, competitive dynamics in the BDKS model are arguably richer because stochastic know-how depreciation implies that a firm can hope to weaken even a well-established rival by depriving the rival of sales.

While most applications of Nash-in-Nash bargaining have been within static models, a small number of papers consider dynamic settings. Lee and Fong (2013) assume period-by-period Nash-in-Nash bargaining in a game where hospitals and insurance companies form networks that change stochastically, while Dorn (2023) shows how Kalai (1977) proportional bargaining facilitates consideration of multiperiod hospital-insurer contracts with index clauses. Yang (2020) embeds Nash-in-Nash bargaining over prices into a dynamic model of innovation in the smartphone device industry and Tiew (2024) considers a dynamic game where duopoly newspapers may bargain over forming or continuing a joint operating agreement. In the current paper bargained prices, through their impact on sales, directly affect sellers' competitiveness and market structure in future periods, which is the central issue in the dynamic competition literature.

The paper is structured as follows. Section 2 outlines the model. Section 3 details the two alternative formulation of the equilibrium equations, our outcome measures and analytic results. Section 4 uses the small state space of the $M = 3$ model to illustrate how reallocating bargaining power changes incentives and strategies. Section 5 provides results and a wider set of policy analyses for the $M = 30$ model. Section 6 concludes. The online Appendices contain proofs, details of methods and additional results.

2 Model

This section presents the model, focusing on where we depart from BDKS. Readers should consult BDKS for additional motivation.

2.1 States and Costs.

There is an infinite horizon, discrete time, discrete state game. The common discount factor is $\beta = \frac{1}{1.05}$.[2]

Sellers. There are two long-lived ex-ante symmetric but differentiated sellers ($i = 1, 2$). i's production cost is $c(e_i) = \kappa\rho^{\log_2(\min(e_i,m))}$, where $e_i = 1, ..., M$ is a commonly observed state variable that tracks the seller's "know-how".[3] The industry state is $\mathbf{e} = (e_1, e_2)$. We will call a seller with higher know-how than its rival the "leader", and its rival the "laggard". A lower value of the progress ratio $\rho \in [0, 1]$ implies stronger learning economies. BDKS assume $m = 15$ and $M = 30$. We will also consider $m = M = 2$ and $m = M = 3$, the smallest state space where a leader can have either a "small" or a "large" advantage.

As described below, one unit will be purchased from one of the sellers in each

[2] As BDKS discuss, one can interpret this discount factor as corresponding to a period length of 1 month, a monthly discount rate of 1 percent and a probability that the industry survives each month of 0.96.

[3] Asker, Fershtman, Jeon, and Pakes (2020), Sweeting, Roberts, and Gedge (2020) and Sweeting, Tao, and Yao (2023) consider dynamic models where serially correlated state variables are private information.

period. e_i evolves, except at the boundaries of the state space, according to

$$e_{i,t+1} = e_{i,t} + q_{i,t} - f_{i,t} \tag{1}$$

where $q_{i,t}$ is equal to one if and only if firm i makes the sale, and $f_{i,t}$ is equal to one (0 otherwise) with probability $\Delta(e_i) = 1 - (1-\delta)^{e_i}$ with $\delta \in [0, 1)$.[4] The probability of forgetting (Δ) is increasing in both δ and e_i. The know-how of a firm that makes a sale can increase by one or stay the same, while the know-how of the firm that does not make the sale will stay the same or decrease by one.

Buyers. Every period a buyer arrives who will purchase exactly one unit from one of the firms. A buyer is assumed to live only one period, and to choose the seller that maximizes her indirect utility, $v - p_i + \sigma\varepsilon_i$, where p_i is the price paid, and the ε_is are i.i.d. private information Type I extreme value payoff shocks. σ parameterizes seller product differentiation.

Bargaining. BDKS assume sellers make simultaneous take-it-or-leave-it price offers. We generalize by assuming that, before the εs are realized, the buyer sends separate agents to each seller. Each agent-seller pair negotiates the price at which the buyer would be able to purchase from that seller. After negotiations are complete, the buyer observes the εs and makes their purchase choice based on the negotiated prices with a purchase possible only if a price was agreed. We make the Nash-in-Nash assumption (Collard-Wexler, Gowrisankaran, and Lee (2019)) that each buyer agent-seller pair takes the price agreed by the other pair as given. We assume that the bargaining weight of the buyer's agent is a parameter τ, with the bargaining weight of the seller $1 - \tau$. If $\tau = 1$, buyers have "all of the bargaining power", and will extract all of the expected surplus from an agreement, whereas we will replicate

[4] At the boundaries of the state space, the evolution is necessarily restricted. For example, when $e_{i,t} = 1$ and $q_{i,t} = 0$, firm i cannot forget ($f_{i,t} = 0$), and when $e_{i,t} = M$ and $q_{i,t} = 1$, firm i has to forget ($f_{i,t} = 1$).

BDKS's equilibria if $\tau = 0$.

The Social Planner. We also compute outcomes given the choice probabilities that would be used by a social planner buyer that maximizes discounted expected total surplus (i.e., buyer utility less production costs) and only knows the current values of the εs.[5]

Parameters. In the text we will assume $\sigma = 1$, $\kappa = 10$ and $\upsilon = 10$, although, with no outside good, the value of υ does not affect equilibrium choice probabilities. Our focus will be on the technology parameters ρ and δ, and, especially, on what happens as τ varies from 0 to 1.

If $\tau \neq 0$, what values of τ are "reasonable" or "relevant"? $\tau = 0.5$, equal buyer-seller bargaining power, is often assumed in applications. As we will illustrate, equilibrium prices, and outcomes, when $\tau = 0.5$ will often not be close to the averages of prices when $\tau = 0$ and $\tau = 1$, as they would often be in the static model, because opportunity costs are endogenous. Alternatively, if one wants to interpret τ as reflecting differences in buyer and seller patience in an alternating offer process taking place in a negotiation, one could view our assumption that buyers are short-lived and sellers long-lived as being most consistent with τs close to zero. Under this interpretation, the main interest will be in how outcomes often change rapidly as τ increases from zero.

3 Equilibrium and Outcomes

The equilibrium concept is symmetric and stationary Markov Perfect Nash equilibrium (MPNE, Maskin and Tirole (2001), Ericson and Pakes (1995), Pakes and McGuire (1994)). We now describe two alternative characterizations of equilibria,

[5] One can view our social planner as analogous to a single long-lived buyer in SJHY's model who faces prices equal to prodution costs each period.

how we measure outcomes and incentives, and results that can be proven analytically.

3.1 Formulation of Equilibrium Conditions for Prices and Values.

BDKS specify equations for equilibrium seller values, $VS^*(\mathbf{e})$, defined at the start of each period, and prices, $p^*(\mathbf{e})$. We extend their formulation to allow for bargaining. Symmetry implies that we only need to define equations for the prices and values of seller 1 (i.e., $p_2^*(e_1, e_2) = p_1^*(e_2, e_1)$ and $VS_2^*(e_1, e_2) = VS_1^*(e_2, e_1)$).

Beginning of period value for firm 1 (VS):

$$VS_1^*(\mathbf{e}) - D_1^*(\mathbf{e})(p_1^*(\mathbf{e}) - c(e_1)) - \sum_{k=1,2} D_k^*(\mathbf{e})\mu_{1,k}^S(\mathbf{e}) = 0, \tag{2}$$

where $\mu_{1,k}^S(\mathbf{e})$ is seller 1's continuation value when seller k makes the sale,

$$\mu_{1,k}^S(\mathbf{e}) = \beta \sum_{\forall e'_{1,t+1}|e_{1,t}} \sum_{\forall e'_{2,t+1}|e_{2,t}} VS_1^*(e'_{1,t+1}, e'_{2,t+1}) \Pr(e'_{1,t+1}|e_{1,t}, k) \Pr(e'_{2,t+1}|e_{2,t}, k), \tag{3}$$

and $\Pr(e'_{i,t+1}|e_{i,t}, k)$ is the probability that i's state transitions from $e_{i,t}$ to $e'_{i,t+1}$ when $q_{k,t} = 1$. Given prices, the probability that $q_{k,t} = 1$, $D_k(\mathbf{e})$, is

$$D_k(\mathbf{e}) = \frac{\exp\left(\frac{v - p_k(\mathbf{e})}{\sigma}\right)}{\exp\left(\frac{v - p_1(\mathbf{e})}{\sigma}\right) + \exp\left(\frac{v - p_2(\mathbf{e})}{\sigma}\right)}. \tag{4}$$

$D_k^*(\mathbf{e})$ is the choice probability given equilibrium prices.

Negotiated prices (p): Our bargaining assumptions imply that, given p_2, $p_1^*(\mathbf{e})$ will be determined as

$$p_1^*(\mathbf{e}) = \arg\max_{p_1} \left(CS(p_1, p_2, \mathbf{e}) - CS(p_2, \mathbf{e})\right)^{\tau} \times ...$$
$$\left(D_1(\mathbf{e})(\mu_{1,1}^S(\mathbf{e}) + p_1 - c(e_1)) + (1 - D_1(\mathbf{e}))\mu_{1,2}^S(\mathbf{e}) - \mu_{1,2}^S(\mathbf{e})\right)^{(1-\tau)} \tag{5}$$

where $CS(p_1, p_2, \mathbf{e}) = \sigma \log\left(\sum_{k=1,2} \exp\left(\frac{v - p_k(\mathbf{e})}{\sigma}\right)\right)$ (i.e., the expected future surplus of the buyer when it is able to choose from both firms). $CS(p_2, \mathbf{e}) = v - p_2(\mathbf{e})$ is the buyer's expected surplus if there is no agreement with seller 1 and seller 2 is the buyer's only option. Equilibrium $p_1^*(\mathbf{e})$ will therefore solve the first-order condition

$$\tau \frac{\partial CS(p_1^*(\mathbf{e}), p_2, \mathbf{e})}{\partial p_1} \left(D_1^*(\mathbf{e})(\mu_{1,1}^S(\mathbf{e}) + p_1^*(\mathbf{e}) - c(e_1)) + (1 - D_1^*(\mathbf{e}))\mu_{1,2}^S(\mathbf{e}) - \mu_{1,2}^S(\mathbf{e})\right) + ...$$
$$(1-\tau)\left(CS(p_1^*(\mathbf{e}), p_2, \mathbf{e}) - CS(p_2, \mathbf{e})\right)\left(D_1^*(\mathbf{e}) + \frac{\partial D_1^*(\mathbf{e})}{\partial p_1}\left(p_1^*(\mathbf{e}) - c(e_1) + \mu_{1,1}^S(\mathbf{e}) - \mu_{1,2}^S(\mathbf{e})\right)\right) = 0, \tag{6}$$

where $\frac{\partial CS(p_1^*(\mathbf{e}), p_2, \mathbf{e})}{\partial p_1} = -D_1^*(\mathbf{e})$ and $\frac{\partial D_1^*(\mathbf{e})}{\partial p_1} = -\frac{D_1^*(\mathbf{e})(1 - D_1^*(\mathbf{e}))}{\sigma}$. Algebraic manipulation shows that this can be simplified to

$$-\tau D_1^*(\mathbf{e})(p_1^*(\mathbf{e}) - \widehat{c_1}(\mathbf{e})) + (1-\tau)\left[\sigma - (1 - D_1^*(\mathbf{e}))(p_1^*(\mathbf{e}) - \widehat{c_1}(\mathbf{e}))\right] \log \frac{1}{1 - D_1^*(\mathbf{e})} = 0. \tag{7}$$

$\widehat{c_1}(\mathbf{e}) = c(e_1) - (\mu_{1,1}^S(\mathbf{e}) - \mu_{1,2}^S(\mathbf{e}))$ is seller 1's opportunity cost of a sale where $(\mu_{1,1}^S(\mathbf{e}) - \mu_{1,2}^S(\mathbf{e}))$ is the increase in the seller's continuation values (equation (3)) when it, rather than its rival, makes the sale.

This first-order condition is the same as BDKS's if $\tau = 0$. If $\tau = 1$, the μ^Ss will equal zero, and $p_1^*(\mathbf{e}) = c(e_1)$ will be the only solution.

3.2 Formulation of Equilibrium Conditions in Terms of Buyer Choice Probabilities.

We can rewrite the equilibrium choice probability equations as

$$\sigma \log\left(\frac{1}{D_1^*(\mathbf{e})} - 1\right) - p_1^*(\mathbf{e}) + p_2^*(\mathbf{e}) = 0. \tag{8}$$

We will now show that prices can be expressed as functions of D_1^*s and parameters, so that the equilibrium can be characterized by equations (8) only.

From equation (7), firm 1's markup over its opportunity cost is

$$p_1^*(\mathbf{e}) - \widehat{c}_1(\mathbf{e}) = \Phi(D_1^*(\mathbf{e})) = \frac{(1-\tau)\sigma \log \frac{1}{1-D_1^*(\mathbf{e})}}{\tau D_1^*(\mathbf{e}) + (1-\tau)(1-D_1^*(\mathbf{e})) \log \frac{1}{1-D_1^*(\mathbf{e})}}. \tag{9}$$

$\mathbf{\Phi}(\mathbf{D_1})$ will denote the stacked vector of markups. Using $\mathbf{Q_k}$ to denote the seller 1 state transition matrix when k makes a sale,

$$\widehat{\mathbf{c_1}} = \mathbf{c_1} - \beta(\mathbf{Q_1} - \mathbf{Q_2})\mathbf{VS_1}, \tag{10}$$

and

$$\mathbf{VS_1} = (\mathbf{I} - \beta \mathbf{Q_2})^{-1} [\mathbf{D_1} \circ \mathbf{\Phi}(\mathbf{D_1})]. \tag{11}$$

where $\circ$ denotes the element-wise product of two vectors. Therefore,

$$\mathbf{p_1} = \mathbf{\Phi}(\mathbf{D_1}) + \mathbf{c_1} - \beta(\mathbf{Q_1} - \mathbf{Q_2})(\mathbf{I} - \beta \mathbf{Q_2})^{-1}[\mathbf{D_1} \circ \mathbf{\Phi}(\mathbf{D_1})]. \tag{12}$$

There are M^2 D_1^*s, but symmetry with no outside good implies $D_1^*(e,e) = \frac{1}{2} \forall e$ and $D_1^*(e,e') = 1 - D_1^*(e',e)$, so we can reduce the problem to $\frac{M(M-1)}{2}$ equations and unknowns. This compares to $2M^2$ equations for prices and seller values in the standard formulation. For $M = 30$, our new formulation reduces the problem from 1,800 equations to 435, and for $M = 3$ the reduction is from 18 equations to only 3.

3.3 Formulation of the Social Planner Problem.

We can find the choice probabilities that maximize the discounted value of expected surplus by thinking of an infinitely lived buyer, with the same discount factor as the sellers, who pays the production cost associated with its choice. In this case,

$$D_1^{SP}(\mathbf{e}) = \frac{1}{1 + \exp\left(\frac{c_1(\mathbf{e}) - c_2(\mathbf{e}) + \mu_2^{SP}(\mathbf{e}) - \mu_1^{SP}(\mathbf{e})}{\sigma}\right)}$$

where μ_i^{SP} is the continuation discounted surplus when seller i is chosen. In vector form,

$$\mu_{\mathbf{2}}^{\mathbf{SP}} - \mu_{\mathbf{1}}^{\mathbf{SP}} = \beta(\mathbf{Q_2} - \mathbf{Q_1})\left(\mathbf{I} - \beta \sum_{k=1,2} \mathbf{D_k^{SP}} \circ \mathbf{Q_k}\right)^{-1} \sum_{k=1,2} \left[\mathbf{D_k^{SP}} \circ \left(\sigma \log \frac{1}{\mathbf{D_k^{SP}}} + v - \mathbf{c_k}\right)\right], \tag{13}$$

where the last term reflects the expected surplus in each state given the choice probabilities. We solve the social planner's choice probabilities using these two sets of equations.

3.4 Outcomes.

We calculate outcome measures assuming the state is (1,1) in $t = 1$.

Concentration. Following BDKS, expected market structure in period t is measured by

$$HHI^t = \sum_{\forall \mathbf{e}} \lambda^t(\mathbf{e}) HHI(\mathbf{e})$$

where

$$HHI(\mathbf{e}) = \sum_{k=1,2} \left(\frac{D_k^*(\mathbf{e})}{D_1^*(\mathbf{e}) + D_2^*(\mathbf{e})}\right)^2$$

and $\lambda^t(\mathbf{e})$ is the probability that a game will be in state $\mathbf{e}$ after t periods. The minimum value of HHI^t is 0.5. One could look at concentration at many different points, but we will focus primarily on HHI^{32} as a measure of medium-run concentration.[6]

[6]For many parameters, HHI^t converges slowly to a long-run value. For example, when $M = 30$, $\rho = 0.75$ and $\delta = 0.023$, $HHI^t \approx 0.5$ for $t > 4,000$, but $HHI^{1,000} \approx 0.6$.

Surplus. With no outside good, v does not affect equilibrium strategies and outcomes, so we exclude it from our surplus measures. TS^t, expected total surplus in period t, is calculated as the expected ε of the purchased good less the production cost. This value will usually be negative. Our focus is on TS^{PDV}, the present discounted value of total surplus. TS^{SP} is the maximized present discounted surplus under a social planner. Consumer surplus is measured as the expected ε less the price paid, and producer surplus as expected price paid less production costs.

3.5 Incentives.

Recall that seller 1's opportunity cost of sale is $\widehat{c_1}(\mathbf{e}) = c(e_1) - (\mu^S_{1,1}(\mathbf{e}) - \mu^S_{1,2}(\mathbf{e}))$ where $c(e_1)$ is the current production cost. Besanko, Doraszelski, and Kryukov (2014) recognize that the dynamic component, $(\mu^S_{1,1}(\mathbf{e}) - \mu^S_{1,2}(\mathbf{e}))$, can be expressed as the sum of two incentives.

Definition 1 *The firm 1 "advantage building" (AB) incentive is* $\mu^S_{1,1} - \mu^S_{1,0}$. *The firm 1 "advantage denying" (AD) incentive is* $\mu^S_{1,0} - \mu^S_{1,2}$.

$\mu^S_{1,0}$ would be seller 1's continuation value if the buyer was to purchase from *neither* seller. This possibility is only hypothetical in the BDKS model, where there is no outside good, but, as we will show, some interesting differences between AB and AD incentives, defined in this way, still arise.

3.6 Analytical Results.

Our focus is on cases where $M = 3$ or $M = 30$, and dynamics, the size of the state space and the possible existence of multiple equilibria make it difficult or impossible to derive analytic results. However, building off BDKS's results, one can show equilibria are unique in some situations.

Proposition 1 *In a model with any* $m \leq M$*,*

1. *if $\tau = 1$, equilibrium prices will equal marginal production costs in all states for all ρ and δ.*

2. *there will be a unique symmetric MPNE when*

   (a) *$\delta = 0$ for all ρ and τ, or*

   (b) *$\tau = 1$ for all ρ and δ.*

Proof. See online Appendix A.1. ■

The social planner problem will also, of course, have a unique solution. Uniqueness when $\delta = 0$ reflects how logit demand implies a unique price equilibrium when continuation values are fixed and how movements through the state space will be unidirectional until the game reaches absorbing state (M, M) so that backwards induction can be applied. When $\tau = 1$, buyers extract all of the expected surplus and prices must compensate sellers for their production costs.

We can prove more novel results when $m = M = 2$ and $\delta = 0$. The $M = 2$ model is tractable because, as $D_1^*(1,1) = D_1^*(2,2) = \frac{1}{2}$ in any symmetric equilibrium and the social planner solution, there is only one choice probability that can vary with τ.[7]

Proposition 2 *For $m = M = 2$ and $\delta = 0$, the unique symmetric equilibrium will have the following properties*

1. *equilibrium $D_1^*(1,2)(\tau) < \frac{1}{2}$ for all τ.*

2. *equilibrium $D_1^*(1,2)(\tau)$ is strictly decreasing in τ.*

3. *for $t \geq 2$, HHI^t is strictly increasing in τ.*

4. *there exists a τ^* such that $TS^{PDV}(\tau^*) = TS^{SP}$, $TS^{PDV}(\tau^*)$ is strictly increasing in τ for $\tau \in (0, \tau^*)$ and strictly decreasing in τ for $\tau \in (\tau^*, 1)$.*

[7]This also facilitates a one-dimensional search for equilibria. We have never found multiplicity with $M = 2$ for any δ.

Proof. See online Appendix A.2. ■

Social welfare will be maximized when $D_1^*(1,2) = D_1^{SP}(1,2)$. The key aspects of the result are that $D_1^*(1,2)(\tau)$, the probability that the laggard will catch up, is monotonically decreasing in τ, so that, as τ increases, the state is more likely to be asymmetric, and that $D_1^*(1,2)(\tau = 0)$ is more than $D_1^{SP}(1,2)$ and $D_1^*(1,2)(\tau = 1)$ is less than $D_1^{SP}(1,2)$. Our computational analysis will show that asymmetric market structures become more likely and efficiency is maximized for τs between 0 and 1 when we consider larger state spaces if LBD effects are significant and forgetting rates are not too high.[8]

3.7 Numerical Methods for Finding Multiple Equilibria.

For given parameters, a single equilibrium can be calculated by solving either formulation of the equilibirum equations, using the Pakes and McGuire (1994) algorithm or, if $\delta = 0$, using backwards induction. To track what happens to equilibrium outcomes when we vary a single parameter, we use BDKS's numerical homotopy method to trace the equilibrium correspondence. We will call a homotopy where a parameter α is varied an "α-homotopy". When we want to identify all equilibria, we also follow BDKS by criss-crossing the parameter space using homotopies, starting new homotopies when additional equilibria are found.

The homotopy method is not guaranteed to find all equilibria in this model, and, in practice, many homotopies stall. For any given set of parameters, our homotopy results must carry the caveat that we could be missing equilibria. However, we are confident in our broad results. We develop an alternative approach for the $M = 3$ model to finidng all equilibria, detailed in online Appendix B.2, that gives identical results. For the $M = 30$ version of the closely-related BDK model, SJHY show that a third approach, which identifies when certain types of equilibria exist, also produces

[8] If $M = 2$, Proposition 2 holds for any $\Delta(2)$ in a slightly changed model where $\Delta(1)$ is set equal to zero (so an $e_{i,t} = 1$ firm will always move to $e_{i,t+1} = 2$ if it makes a sale).

results consistent with an extensive homotopy search.

4 Bargaining, Concentration and Welfare with $M = 3$

A model with $m = M = 3$ is too simple to represent any industry, but it allows us to follow how bargaining power affects every price and choice probability.

4.1 Bargaining and the Existence of Multiple Equilibria.

Multiple equilibria are widely understood to complicate analysis of comparative statics and counterfactuals. Uniqueness is guaranteed when $\tau = 1$, but it is unclear *a priori* whether multiple equilibria will be more common when sellers set prices or when bargaining power is split between buyers and sellers.

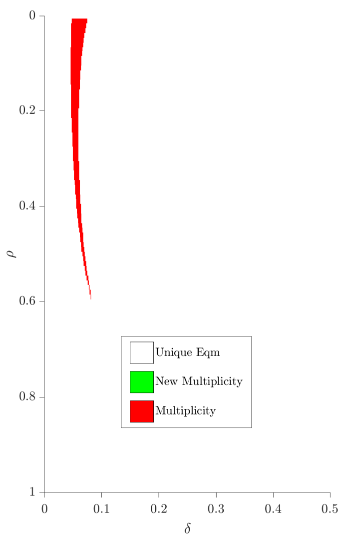
We identify equilibria by running sequences of ρ and δ homotopies for discrete values of τ in 0.05 steps. The set of identified equilibria are identical when we use our alternative method. The colored areas in Figure 1 indicate (ρ, δ) combinations with multiple equilibria (we never find more than three) for τs up to 0.2. For $\tau \geq 0.25$ and $\delta \geq 0.1$ equilibria are always unique. While there are a few technology parameters, marked in green, where multiple equilibria are introduced when τ increases, the clear pattern is that allocating more bargaining power to buyers tends to eliminate multiplicity.

4.2 Bargaining, Equilibrium Prices and Outcomes for $\rho = 0.3$ and $\delta = 0.03$.

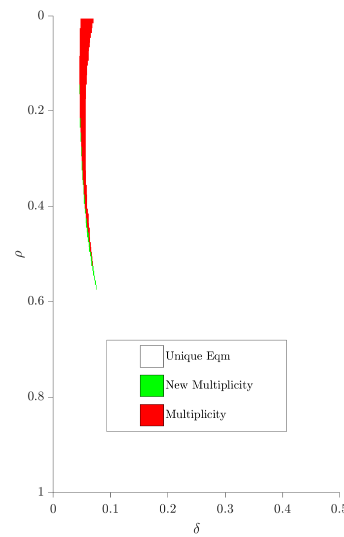
Figure 2 shows how choice probabilities, prices, concentration and surplus change when we trace equilibria using a τ-homotopy for $\rho = 0.3$ and $\delta = 0.03$. Equilibria are unique, for all τ, for these technology parameters. Online Appendix Table C.2 shows the exact prices and sale probabilities in each state, and the distributions of states after 4 and 32 periods, for $\tau = 0$, $\tau = 1$ and the social planner solution. $c(e_i) = 10$,

Figure 1: Multiplicity of Equilibria in the $M = m = 3$ Model. White = unique equilibrium. Green = multiple equilibria for τ and uniqueness for $\tau - 0.05$. Red = multiple equilibria for τ and (for $\tau \geq 0.05$) also for $\tau - 0.05$. For $\tau = 0.2$, there is a very small red area for $\rho < 0.05$ and $\delta \approx 0.058$. We do not identify any multiplicity for $\tau \geq 0.25$, using steps of 0.05, or for $\delta > 0.5$ for all (ρ, τ). The figures were constructed using homotopies that criss-crossed the (ρ, δ) parameter space at least five times.

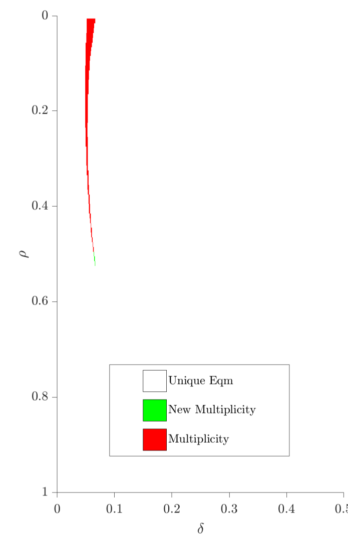
(a) $\tau = 0$



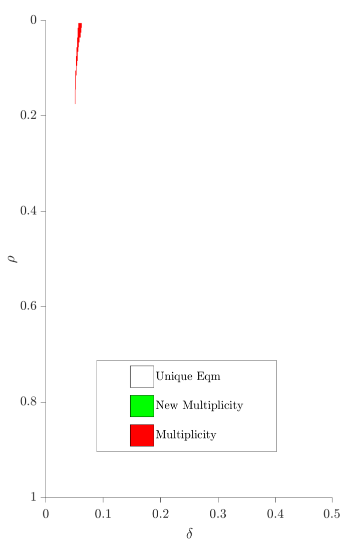
(b) $\tau = 0.05$



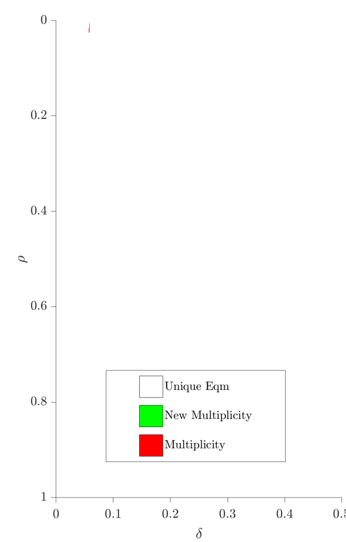
(c) $\tau = 0.10$



(d) $\tau = 0.15$



(e) $\tau = 0.20$



3 and 1.483 for $e_i = 1, 2, 3$ so that LBD effects are substantial. $\Delta(e_i) = 0.03$, 0.059 and 0.087 so that high know-how firms are unlikely to experience depreciation in any given period even if they do not make a sale.

It is useful to begin by understanding the difference between the social planner's purchase strategy (choice probabilities indicated by the horizontal dashed lines in Figure 2(a)) and the choices of a myopic buyer facing prices equal to production costs, the equilibrium when $\tau = 1$.

When $\tau = 1$ a myopic buyer makes choices based on current production costs and current εs. An $e_i = 1$ laggard has such a large cost disadvantage in asymmetric states that it is very unlikely to make a sale. This results in the industry state being (3,1) with probability 0.996 after 32 periods, and the seller with the lower ε making the sale with probability close to $\frac{1}{2}$ from the second period onwards.

The social planner recognizes that a second low cost seller will increase expected future εs. The social planner's willingness to invest in a second low cost option is illustrated by how $D_1^{SP}(1,3) > D_1^{SP}(1,2) > D_1^*(1,2)(\tau = 1)$, even though $c_2(1,3) < c_2(1,2)$, and $D_1^{SP}(2,3) > \frac{1}{2} >> D_1^*(2,3)(\tau = 1)$. The social planner solution implies moderate concentration after four periods, but the industry will be in the highest surplus state, (3,3), after 32 periods with probability 0.789.

Besanko, Doraszelski, and Kryukov (2019a) decompose efficiency losses into a loss of surplus in each state relative to the social planner outcome, which they label the "PR distortion", and the loss because states are not visited with the probabilities that the social planner would choose ("MS distortion").[9] Figure 2(e) (right axis) shows

[9]Define

$$DWL_\beta^{PR} = \sum_{t=0}^{\infty} \beta^t \sum_{\mathbf{e}} \lambda_t(\mathbf{e}) \left[TS^{SP}(\mathbf{e}) - TS(\mathbf{e})\right]$$

and

$$DWL_\beta^{MS} = \sum_{t=0}^{\infty} \beta^t \sum_{\mathbf{e}} \left[\lambda_t^{SP}(\mathbf{e}) - \lambda_t(\mathbf{e})\right] TS^{SP}(\mathbf{e}).$$

where $TS(\mathbf{e})$ and $TS^{SP}(\mathbf{e})$ represent the value of expected εs less production costs given equilibrium and social planner choice probabilities in state $\mathbf{e}$ respectively, and $\lambda_t(\mathbf{e})$ and $\lambda_t^{SP}(\mathbf{e})$ are the probabilities the game is in state $\mathbf{e}$ after t periods. $DWL_\beta^{PR} + DWL_\beta^{MS}$ is the difference between the values of these two distortions. When $\tau = 1$, the buyer maximizes surplus in each state, whereas the social planner's choice may forgo some current surplus. Therefore, DWL_{β}^{PR} is negative, but DWL_{β}^{MS} is positive and large as state (3,3) is rarely visited. The sum of these distortions indicates that TS^{PDV} is lowered by around 2.4 compared to the social planner optimum.

Now consider the equilibrium when $\tau = 0$. Sellers charge markups over their opportunity costs and their opportunity costs reflect dynamic incentives (i.e., expected future profits), so that DWL_{β}^{PR} is positive. However, relative to $\tau = 1$, these effects may move sale probabilities and state transitions in a socially desirable direction. For example, $D_1^*(1,2)$ is increased by the laggard having larger dynamic incentives (shown below), lowering its opportunity cost of sale, and the leader having a larger markup. In fact, $D_1^*(1,2)(\tau = 0) > D_1^{SP}(1,2)$ and $D_1^*(1,3)(\tau = 0) > D_1^{SP}(1,3)$, and $D_1^*(2,3)(\tau = 0) > \frac{1}{2}$ even though it is slightly lower than $D_1^{SP}(2,3)$. These D_1^*s imply that the state is more likely to move from (2,1) to (2,2) (rather than (3,1)) than the social planner would choose, but they also imply that state (3,3) will be reached more quickly. As a result DWL_{β}^{MS} is negative, and the decrease in TS^{PDV} from the social planner optimum is small.

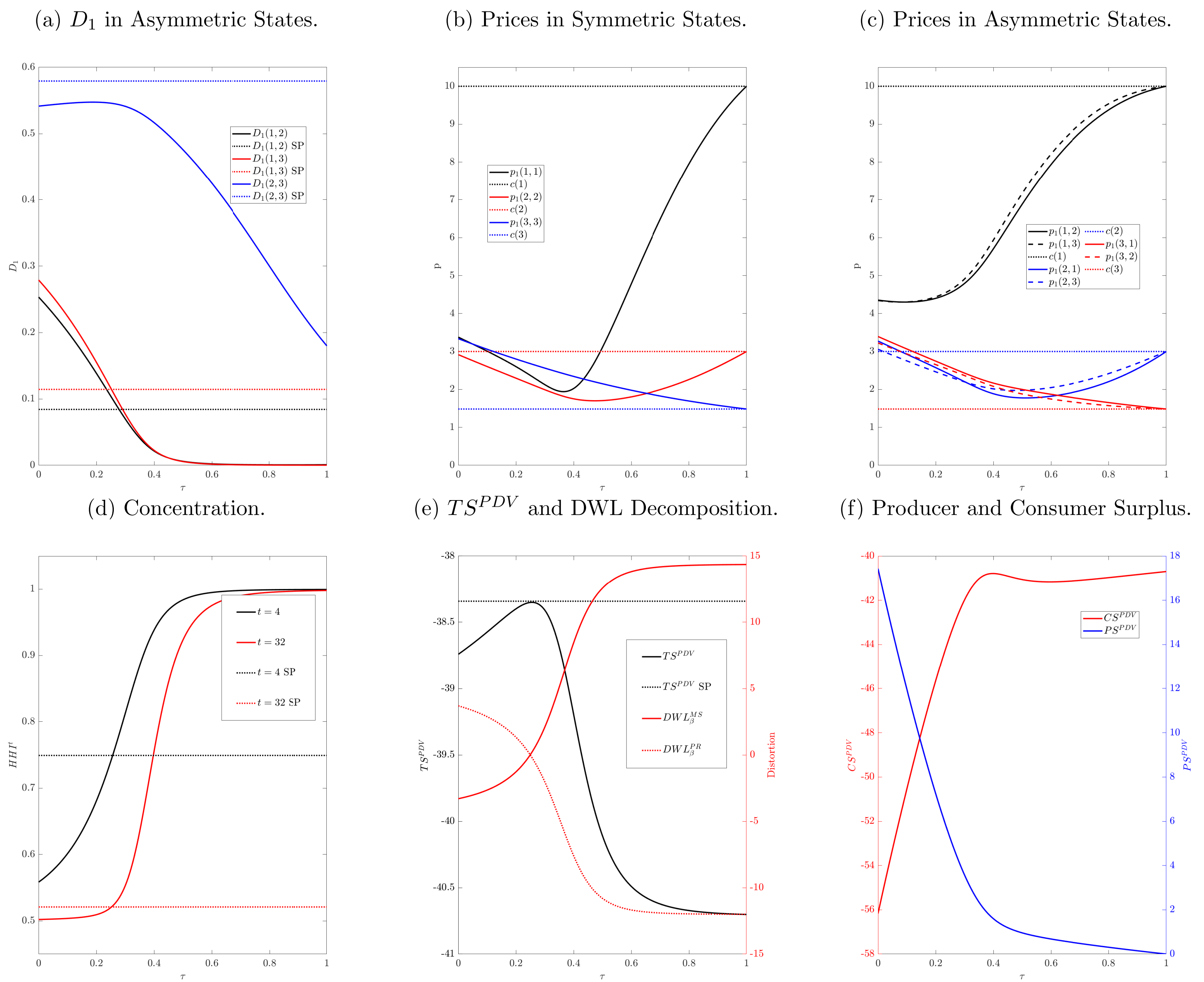
Looking between these polar cases, we see that HHI^4 and HHI^{32} (concentration) increase monotonically in τ, and that TS^{PDV} is maximized for $\tau \approx 0.25$. The maximized surplus is slightly lower than the social planner achieves as there is no τ where all three asymmetric state D_1^*s match their social planner values (in particular, no equilibrium gives the laggard a high enough probability of selling in state (2,3)). Concentration and consumer surplus (panel (f)) increase significantly between 0 and 0.5, with producer surplus falling sharply, and several prices, including $p^*(1,1)$ and $p^*(2,2)$, are non-monotonic in τ.

Cabral and Riordan (1994) reason that one firm will emerge as dominant when prices that satisfy the properties of "increasing dominance" (ID, lower cost firms

---

social planner and equilibrium surplus. Besanko, Doraszelski, and Kryukov (2019a) also consider a third component related to entry and exit which is not relevant in the BDKS model.

Figure 2: The Effects of Bargaining Power Parameter (τ) on Equilibrium Outcomes and Welfare for $\rho = 0.3$ and $\delta = 0.03$. SP = social planner outcome, $c(e_i)$ are production costs for a firm with know-how e_i. Homotopies and our alternative method establish that there is a unique equilibrium for every τ. DWL = Deadweight loss.



always set lower prices) and "increasing increasing dominance" (IID, $p_1^*(\mathbf{e}) - p_2^*(\mathbf{e})$ decreases in e_1). Prices equal to costs satisfy these properties, and concentration is excessive when $\tau = 1$, whereas $\tau = 0$ equilibrium prices do not satisfy these properties (e.g., $p_1^*(2,1) < p_1^*(3,1)$) and concentration is too low. However, consistent with an observation of BDKS, ID and IID are not necessary for concentration to be either high or excessive: for example, for $0.25 < \tau < 0.445$, prices do not satisfy these properties even though HHI^4 is close to one and much larger than the social planner would choose.

4.3 Bargaining Power on Dynamic Incentives.

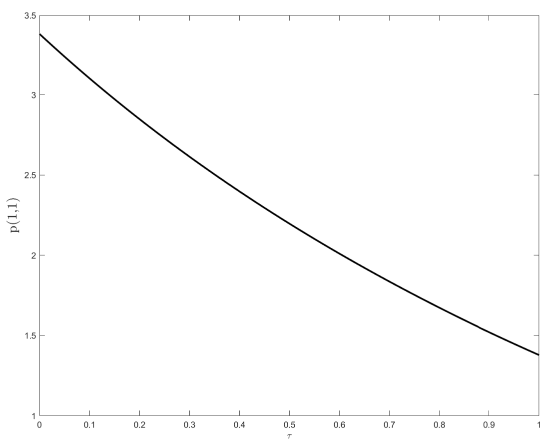
We now explore the relationship between τ and the equilibrium values of dynamic incentives. This relationship will explain why several prices have U-shaped relationships with τ and why concentration and some measures of surplus (including TS^{PDV} in the $M = 30$ model) change quickly as τ increases from zero.

Figure 3 illustrates different effects of varying τ. We focus primarily on state (1,1) as there is only one price. Panel (a) shows how $p(1,1)$ would change with τ when continuation values are held fixed at their $\tau = 0$ values. This isolates the markup reduction effect that occurs in static models. $p(1,1)$ falls monotonically, and close to linearly, to equal opportunity costs when $\tau = 1$.

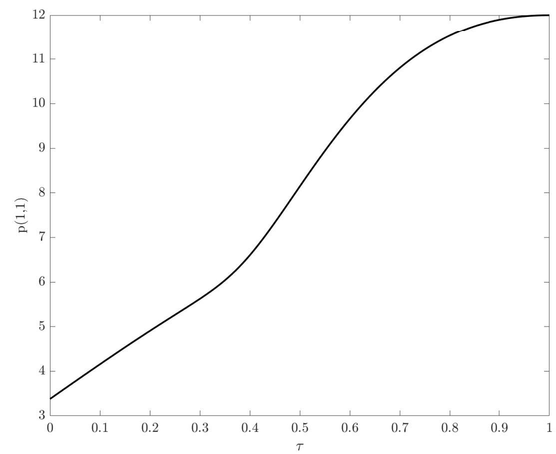
The source of the non-monotonicity therefore lies in how continuation values vary with τ. One can consider τ as having two effects on continuation values. First, fixing industry evolution, increasing τ reduces seller continuation values, and shrinks the difference between the value of making a sale and the rival making a sale, as the share of surplus in every future negotiation is reduced. Panel (b) isolates this effect by showing the equilibrium price that sellers would set (i.e., $\tau = 0$ static game) when $\tau = 0$ continuation values are multiplied by one minus the τ value on the x-axis. As future profits shrink, a seller's opportunity cost rises towards its production cost, and the take-it-or-leave-it $p(1,1)$ rises monotonically.

Figure 3: Prices and Incentives as a Function of τ for $\rho = 0.3$ and $\delta = 0.03$ in the $M = m = 3$ Model.

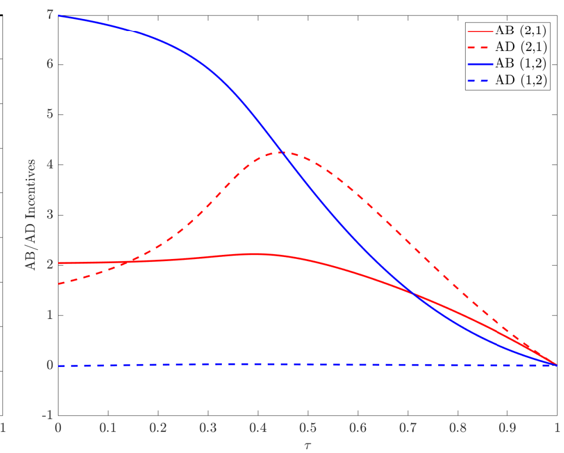
(a) Eqm. (1,1) Price Given $\tau = 0$ Continuation Values



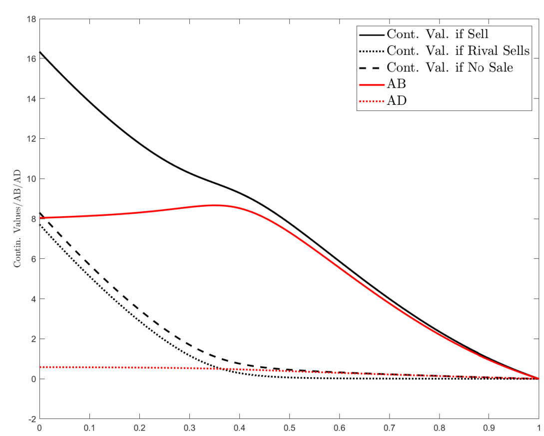
(b) $\tau = 0$ Eqm. (1,1) Price Given $(1-\tau)\times$ $\tau = 0$ Continuation Values



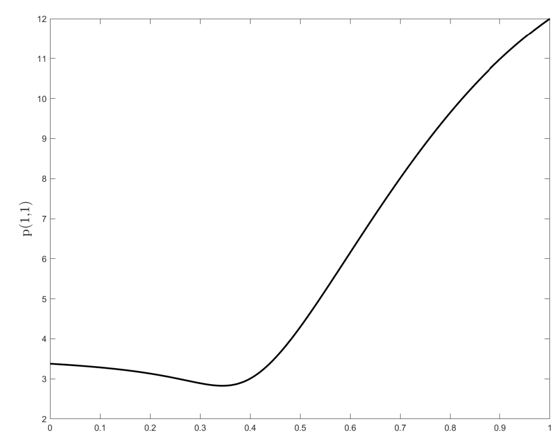
(c) AB and AD Incentives for $\mathbf{e}$ =(2,1) and (1,2)



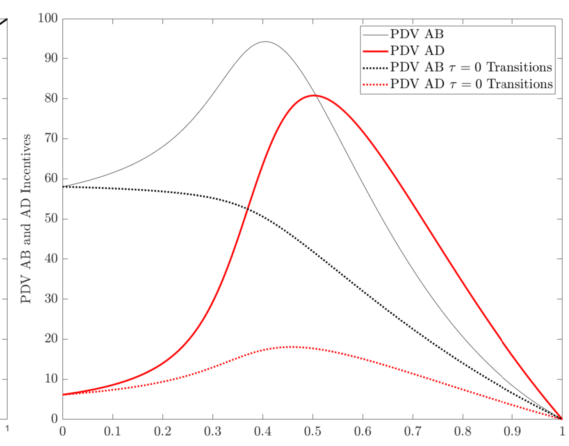
(d) Eqm. (1,1) Contination Values and AB/AD Incentives



(e) $\tau = 0$ Eqm. (1,1) Price Given τ Eqm. Continuation Values..



(f) Eqm. PDV AB/AD Incentives



Second, reallocating bargaining power will change how market structure evolves. In particular, the static markup reduction effect will increase the sale probability of the firm with the larger markup. If this firm is the leader, which it is in all asymmetric states when $\tau = 0$ for our parameters, then this will tend to make leads last longer. If leads would be short-lived when $\tau = 0$, this lead-lengthening effect can cause the relative value of acquiring or preserving a lead to increase in τ, so the leader's opportunity cost falls, lowering the leader's price.

To illustrate, consider what happens in state (2,1), which is the state that seller 1 in state (1,1) wants to reach. A firm in state (2,1) expects its lead to last for 5.4 periods when $\tau = 0$. This increases to 11.3 and 196 periods when $\tau = 0.25$ or 0.5 respectively. Correspondingly, a firm in state (2,1) loses its lead within three periods with probability 0.378 when $\tau = 0$, and with probabilities 0.168 and 0.009 when $\tau = 0.25$ and $\tau = 0.5$ respectively.

Panel (c) shows AB and AD incentives in state (2,1) for both sellers. This panel also reveals that changes in the laggard's dynamic incentives also make the leader more likely to sell in (2,1). Adding its AB and AD incentives together, the laggard has a larger dynamic incentive than the leader when $\tau = 0$, partially offsetting the laggard's production cost disadvantage. However, as τ increases, the leader's AD incentive increases as its lead is expected to last longer if its rival does not make a sale, while the reduction in equilibrium $p^*(2,2)$ reduces laggard's AB incentive. The leader's dynamic incentive exceeds the laggard's for $\tau \geq 0.25$.

Panel (d) shows that all three equilibrium $\mu_1^S(1,1)$ continuation values decline monotonically in τ increases but that the equilibrium AB incentive increases until $\tau \approx 0.4$, and the opportunity cost of sale decreases until $\tau \approx 0.35$. Panel (e) repeats the exercise in panel (b) but using these equilibrium continuation values. The changes in continuation values (i.e., dynamic effects) now lower the price, reinforcing the static markup effect, until $\tau \approx 0.37$. This explains why $p^*(1,1)$ initially declines in τ (Figure 2(b)). For larger τ, the shrinkage of continuation values, and the implied increase in opportunity costs, is the stronger dynamic effect of increasing τ. $p^*(1,1)$ therefore increases. While changes in $p^*(1,1)$ do not change concentration, the reductions in the leader's markup and its opportunity cost increase the leader's probability of making sales in (2,1) and (3,1) (Figure 2(a)) which does cause concentration to increase.

The solid lines in panel (f) show that the discounted expected value of sellers' combined AB and AD incentives given equilibrium play. Both AB and AD incentives increase and then decrease in τ. The dotted lines show what the discounted values would be if we use the equilibrium values of the incentives in each state but assume $\tau = 0$ equilibrium state transitions. The differences between the solid and dotted lines highlight how the non-monotonicity in dynamic incentives is accentuated by how play shifts towards states such as (3,1), where dynamic incentives are larger, as τ increases from zero.

Generalization to Other Technology Parameters. Online Appendix C.2 provides an analysis of how these patterns generalize to other (ρ, δ) technology parameters. The pattern that concentration increases with τ holds for most parameters, but is not universal. Online Appendix C.3 shows an example with very limited LBD, where a twist in the equilibrium correspondence associated with multiple equilibria causes concentration to fall, at a low level, as τ increases from zero. Other examples, with significant LBD, involve very high concentration which declines slightly as τ increases from 0.5 to 1.

The other patterns that we have described (e.g., TS^{PDV} and the discounted value of dynamic incentives having inverted-U relationships with τ) are observed when $\rho < 0.8$ (i.e., costs when $e_i = 3$ are no more than 70% of the costs when $e_i = 1$) and $\delta \leq 0.05$. For $\delta > 0.05$, forgetting is likely and a laggard with infrequent sales is unlikely to catch up, so that concentration is high and leads tend to be long-lasting even when $\tau = 0$. The effect of shrinking continuation values therefore tends to dominate the lead-lengthening effect and the values of dynamic incentives are maximized when $\tau = 0$. For $\delta > 0.1$, it is too costly for the social planner to maintain two low cost sellers, and the planner's strategy shifts towards maximizing current surplus. As a result, efficiency is maximized for τs close to, or equal to, 1.

Outside Option and Product Differentiation. We have assumed that $\sigma = 1$ and that buyers have no outside option. Online Appendix C.4 relaxes these assumptions. Outcomes are quite robust to moderate variation in these assumptions, such as allowing an outside good that is only really competitive when the sellers have the lowest know-how. However, significant reductions in product differentiation (lower σ) result in more concentrated equilibrium outcomes, and leads that are expected to last longer, for all τ. Consistent with the logic described so far, this tends to mean that dynamic incentives will be strongest when $\tau = 0$.

4.4 Subsidies.

We consider schemes that implement socially optimal choice probabilities by taxing or subsidizing sales by the laggard.[10] Given socially optimal choice probabilities (D_1^{SP}), we can solve for M^2 prices and $\frac{M(M-1)}{2}$ subsidies (s_1, negative for a tax) using M^2 equations

$$p_1(\mathbf{e}) - p_2(\mathbf{e}) = \sigma \log\left(\frac{1}{D_1^{SP}(\mathbf{e})} - 1\right), \tag{14}$$

and $\frac{M(M-1)}{2}$ stacked equations

$$\mathbf{p_1} + \mathbf{s_1} = \mathbf{\Phi}(\mathbf{D_1^{SP}}) + \mathbf{c_1} - \beta(\mathbf{Q_1} - \mathbf{Q_2})(\mathbf{I} - \beta\mathbf{Q_2})^{-1}[\mathbf{D_1^{SP}} \circ \mathbf{\Phi}(\mathbf{D_1^{SP}})]. \tag{15}$$

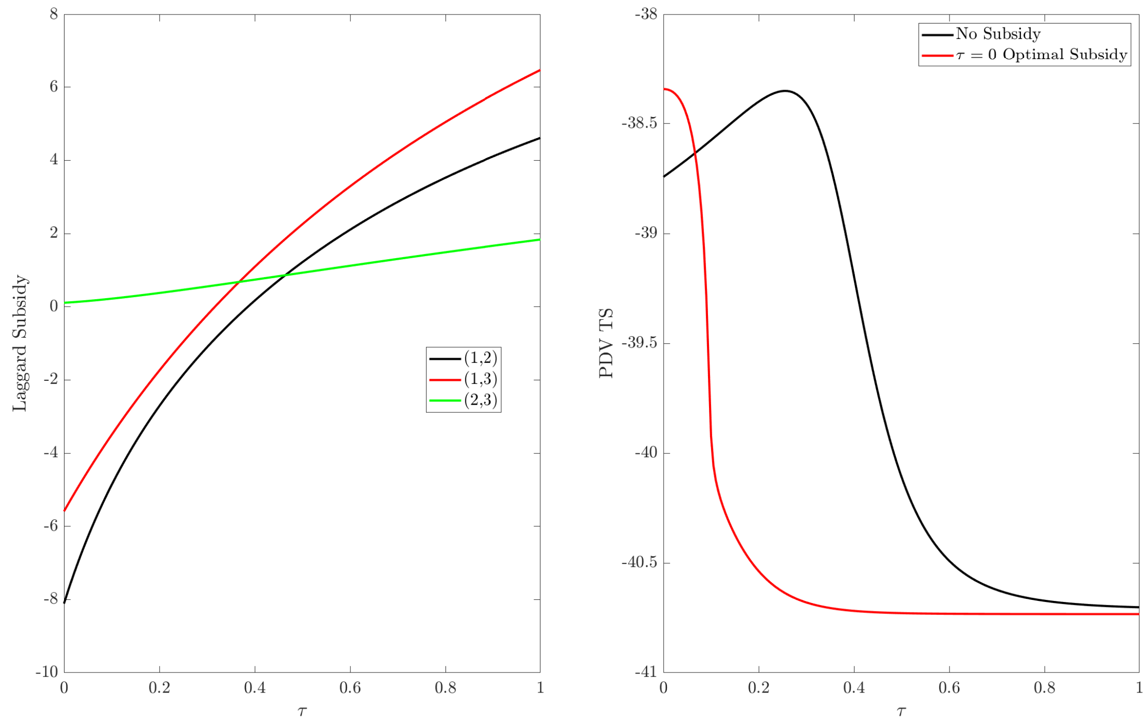
Linearity in p and s implies that only one scheme can implement the social optimum but this scheme could support equilibria that are not optimal (see $M = 30$ example

[10] With no outside good, sale probabilities in symmetric states are efficient so it is natural to consider schemes that impose taxes or subsidies in asymmetric states. Taxes and subsidies are fully passed through to buyers when $\tau = 1$.

Figure 4: Subsidy Schemes and Welfare as a Function of τ for $\rho = 0.3$ and $\delta = 0.03$ in the $M = m = 3$ Model.

(a) Optimal Subsidy Levels

(b) TS^{PDV} Under $\tau = 0$ Optimal Subsidies.



below).

Figure 4(a) shows the optimal subsidies for our example technology parameters. Laggard sales are taxed in states $(1, 2)$ and $(1, 3)$ when $\tau = 0$, but they are subsidized in all states when $\tau > 0.4$. Further calculations show that the optimal scheme can have non-monotonic effects on some outcomes: for example, relative to a no subsidy scenario, discounted consumer surplus increases when $\tau \approx 0$ and $\tau > 0.6$, but it falls slightly for $\tau \approx 0.3$.

We can also ask how welfare would change for different τs if a subsidy scheme is designed assuming $\tau = 0$. $\tau = 0$ would be the natural assumption for a policy designer to make based on the existing literature. Figure 4(b) shows that the scheme would lower welfare, relative to having no scheme at all, for any τ that is larger than

0.06, a result that implies that buyers only have to have limited bargaining power for the subsidy policy to be welfare-reducing. The intuition for this result is that the $\tau = 0$ scheme aims to increase industry concentration which, with no subsidies, would be close to optimal for τs that are not too large and already excessive for higher τ. Online Appendix C.5 shows that $\tau = 0$ subsidies would also lower welfare if $\tau = 0.2$ for a wide range of technology parameters with low δs. For higher δs, some of the $\tau = 0$ subsidies are so large that we cannot solve for equilibria with alternative τs.

5 $M = 30$ Model

BDKS use the $m = 15$ and $M = 30$ model as a potentially stylized representation of a real-world industry. We describe how the features that we observe in the $M = 3$ model carry over to this larger state space model before examining optimal subsidies and some alternative policy counterfactuals.

Our policy analysis will assume $\rho = 0.75$ (so that costs for $e_i \geq 15$ are just over 30% of their $e_i = 1$ value) and $\delta = 0.023$ (implying $\Delta(30) \approx 0.5$), which we will refer to as the illustrative technology parameters. When considering what happens with no policies, we look across $0.6 \leq \rho \leq 1$, reflecting the range of empirical progress ratios identified in the literature survey of Ghemawat (1985), and $0 \leq \delta \leq 0.2$. $\delta = 0.2$ implies almost certain depreciation of know-how for $e_i \geq 20$.

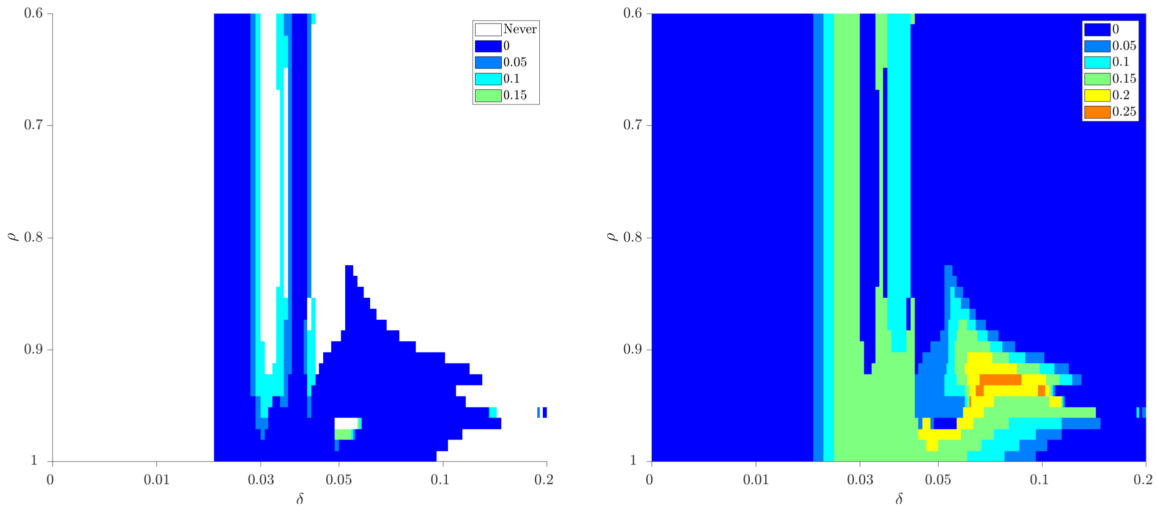
5.1 Equilibria with No Policies.

The qualitative results from our $M = 3$ analysis carry over to the larger state space. First, multiple equilibria are eliminated once τ is even moderately high. Based on a homotopy analysis to identify equilibria for τ values in 0.05 increments, Figure 5(a) shows the smallest τs for which multiple equilibria are identified, and (b) shows the τ values for which equilibria are unique for all larger τs. Multiplicity exists for a wider range of technologies than in the $M = 3$ model when $\tau = 0$, but we still find that all

Figure 5: Multiplicity in the $M = 30$ Model. In panel (a) the white regions correspond to parameters for which multiplicty is never identified.

(a) Smallest Values of τ where Multiple Equilibria Are Identified.

(b) Smallest Values of τ' where Equilibria are Unique For All $\tau \geq \tau'$.



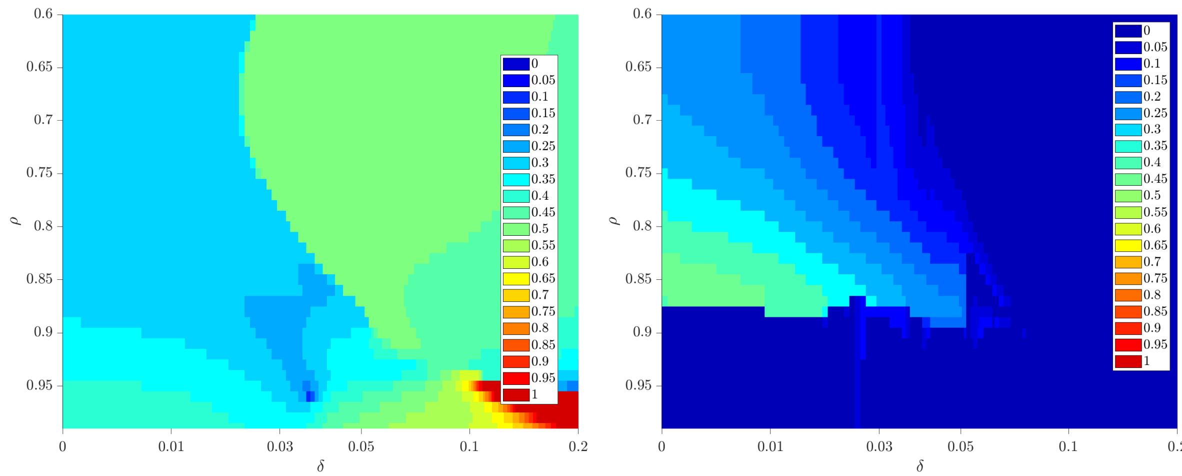
equilibria are unique once $\tau \geq 0.25$. We have undertaken further analysis to try to identify if there is a pattern as to the types of $\tau = 0$ equilibria (e.g., the nature of pricing strategies) that are on homotopy paths that ultimately reach $\tau = 1$ but we have not been able to identify a general pattern. See additional discussion in online Appendix C.3.

Second, as we detail in online Appendix D.2, the pattern that equilibrium concentration increases with τ, at least until it reaches a very high level, is fairly general when LBD effects are not too small.[11] HHI^{32} is lower than the social planner would choose when $\tau = 0$ for most technologies. For $\rho < 0.85$ and $\delta < 0.03$, equilibrium HHI^{32} is low when $\tau = 0$, very high when $\tau = 1$, and close to the level that the social planner would choose between $\tau = 0.25$ and $\tau = 0.5$. Consistent with this feature, Figure 6(a) shows that τs between 0.2 and 0.5 maximize surplus for a wide range of technologies.

[11]There are parameters with very limited LBD and moderate-to-high forgetting probabilities where HHI^{32} is above the social planner level, and where it falls when τ increases, before following an upward path. See online Appendix D.3 for an example.

Figure 6: Values of τ Maximizing TS^{PDV} and the PDV of Seller Dynamic Incentives (the sum of AB and AD incentives) in the $M = 30$, $m = 15$ model. We take the maximum value across equilibria when multiple equilibria exist for given τ.

(a) τ Maximizing TS^{PDV} (b) τ Maximizing PDV AB+AD Incentives

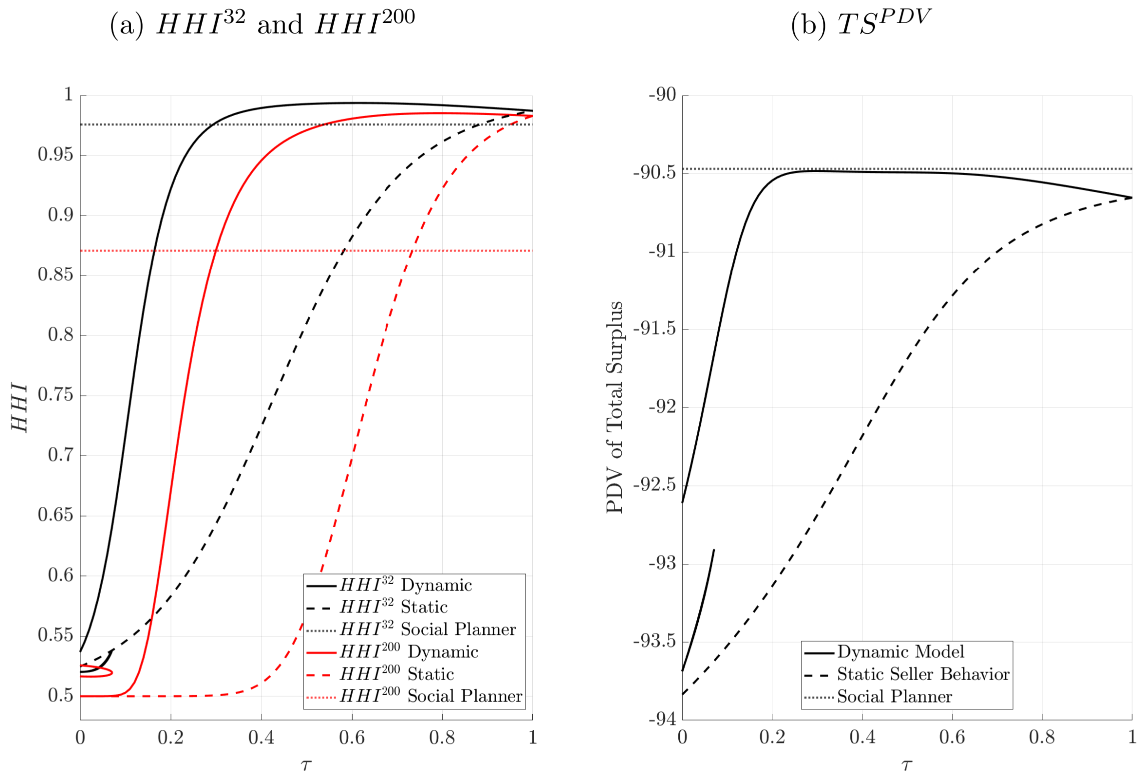


Third, for $\delta < 0.03$ and $\rho < 0.85$, we also see that τs between 0.2 and 0.5 maximize sellers' dynamic incentives (Figure 6(b)). The increase in dynamic incentives as τ increases from zero is associated with leads lasting significantly longer: for example, for the illustrative technology parameters, the PDV of dynamic incentives increases from around 80 when $\tau = 0$ to around 170 when $\tau = 0.2$. This is associated with the number of periods that a firm in state (2,1) expects its lead to last increasing from 90 to 300, and the probability that it loses its lead within three periods falling from 0.119 to 0.009.[12]

The increases in dynamic incentives also leads to a divergence between predicted outcomes in the dynamic model and the outcomes that would be predicted by a model where sellers are assumed to behave statically even though know-how can evolve. Figure 7 illustrates this divergence by showing the values of HHI^{32}, HHI^{200} and TS^{PDV} along τ-homotopy paths for these two types of equilibrium given the illustrative technology parameters. Equilibria will always be unique when sellers

[12]These statistics are associated with the $\tau = 0$ equilibrium on the τ-homotopy path that stretches from $\tau = 0$ to $\tau = 1$ (see Figure 7).

Figure 7: Concentration and Welfare Outcomes with Dynamic and Static Equilibrium Seller Behavior as a Function of τ for $\rho = 0.75$ and $\delta = 0.023$ with no policies.



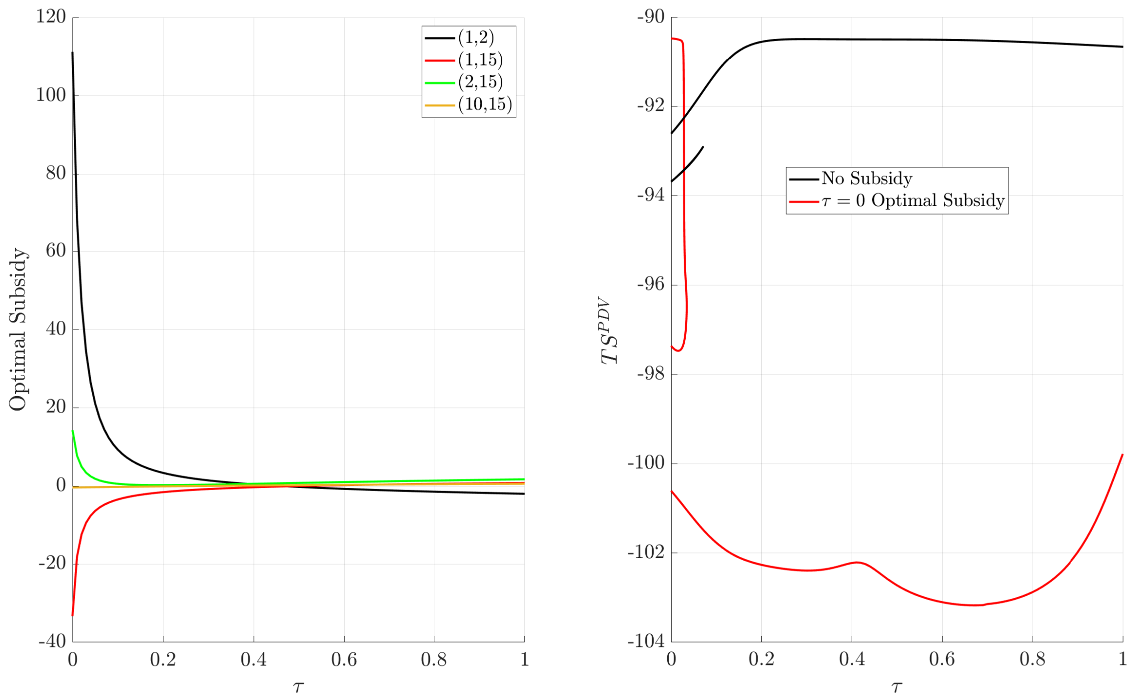
ignore dynamic incentives, and they will coincide with the dynamic equilibrium when $\tau = 1$. When $\tau = 0$, concentration levels in the three dynamic equilibria and the static equilibrium are similar, but equilibrium concentration increases much more quickly as τ increases in the dynamic model, and HHI^{32} actually falls, by a small amount, as τ increases from 0.5 to 1. The difference in TS^{PDV} given dynamic and static behavior is maximized for $\tau \approx 0.2$, where the dynamic equilibrium gets very close to the planner optimum.

5.2 Subsidy Policies.

We now turn to the policy analysis, by considering optimal subsidies in the $M = 30$ model, before considering alternative policies to promote competition. We will focus on the illustrative technology parameters, as possibly reflecting an industry with

Figure 8: Optimal Subsidies and Welfare under $\tau = 0$ Subsidies as a Function of τ for the Illustrative Technology Parameters. The subsidy is given to the laggard when it makes a sale, and a negative number is a tax.

(a) Optimal Subsidies for Selected States (b) Total Surplus Under $\tau = 0$ Subsidies



fairly strong LBD effects and some depreciation. We have found that some other technologies support many equilibria for alternative policies. Online Appendix D.3 shows some policy results for technology parameters with limited LBD.

Figure 8(a) shows optimal laggard subsidies in four different states as a function of τ. The optimal subsidy in state (1,2) and the optimal tax on the laggard in state (1,15) change dramatically as τ increases from zero, illustrating how small deviations from the price-setting assumption would need to be accounted for in policy design.

The direction of the $\tau = 0$ subsidies may also appear counterintuitive. When $\tau = 0$, equilibrium concentration is lower than the social planner would choose. So why is it optimal to subsidize sales by the laggard?

The reason lies in how the future evolution of market structure affects dynamic incentives. The social planner's evolution, which is being implemented, will lead to a firm in (3,1) being likely to preserve its lead for a long-time. Under this evolution, the dynamic incentives of a $\tau = 0$ leader in state (2,1) would be extremely large (its AB incentive is 127.3 and its AD incentive is 80.7) without subsidies, implying the leader would make a sale with a probability that is actually much higher than the social planner wants. Therefore, a laggard subsidy is appropriate. Similarly, because the social planner's choices would lead the laggard to make most of the sales once the leader has know-how above m, the leader's dynamic incentives in (1,15) would be much smaller than the laggard's so that a tax on the laggard is appropriate.

The red lines in Figure 8(b) show TS^{PDV} under the subsidies that would be optimal if $\tau = 0$. The black line shows TS^{PDV} in the no subsidy equilibrium. When τ is very small, $\tau = 0$ optimal subsidies support multiple equilibria with different welfare levels. The only equilibrium when $\tau \geq 0.04$ implies that $\tau = 0$ optimal subsidies lead to lower welfare than with no policy, further illustrating the importance of knowing the value of τ when determining policies.

5.3 Policies to Increase Competition.

We use the illustrative technology parameters to analyze policies that might be expected to "promote competition" (i.e., more symmetric market structures). Note that concentration is too low, at least initially, for these parameters when $\tau = 0$, so one might think of our analysis as primarily trying to understand the costs of the types of policies that have been suggested in industries where dynamic competition is important, whereas, if $\tau = 0.5$, one might read our analysis as assessing whether the welfare benefits of increased seller symmetry are offset by the distortions that the policies will introduce. We will also see that, for higher values of τ, some of the policies can also have unexpected effects on market structure.

5.3.1 Alternative Policies.

Restriction on Market Concentration. Benkard (2004), analyzing the market for wide-bodied commercial aircraft, considers a counterfactual where a limit is imposed on the market share of the largest firm in a given quarter.[13] In our duopoly single-buyer-per-period model, we implement the share restriction as a soft constraint by assuming that a market leader has to pay a compliance penalty of $\chi \times \max\{0, D_i - \psi\}^2$ whenever its sale probability is above a threshold $\psi > 0.5$. As χ increases, it becomes more costly for a firm to have a high market share.[14] We calculate TS^{PDV} assuming that compliance penalties are not costs to society. The results presented in the paper assume that $\psi = 0.75$, so that policy maker aims that the leader should be no more than three times as likely as the laggard to make a sale, and $\chi = 50$, a value that is large enough that the constraint is rarely breached in equilibrium. See online Appendix D.4 for an analysis of how concentration and TS^{PDV} change with the assumed level of χ.

Incorporating this penalty, the first-order condition for the negotiated price becomes

$$-\tau\left[D_1^*(\mathbf{e})(p_1^*(\mathbf{e}) - \widehat{c}_1) - \chi \max\{0, D_1^*(\mathbf{e}) - \psi\}^2\right] + (1-\tau)\log\left(\frac{1}{1-D_1^*(\mathbf{e})}\right) \times \\ \left[\sigma - (1-D_1^*(\mathbf{e}))(p_1^*(\mathbf{e}) - \widehat{c}_1) + 2\chi(1-D_1^*(\mathbf{e}))\max\{0, D_1^*(\mathbf{e}) - \psi\}\right] = 0, \tag{16}$$

and the equation for the seller's value becomes

$$VS_1^*(\mathbf{e}) - D_1^*(\mathbf{e})\left(p_1^*(\mathbf{e}) - c(e_1)\right) - \sum_{k=1,2} D_k^*(\mathbf{e})\mu_{1,k}^S(\mathbf{e}) - \chi\max\{0, D_1^*(\mathbf{e}) - \psi\}^2 = 0. \tag{17}$$

[13]While absolute restrictions on market shares are rare, market shares can play an important role in determining potential liability for actions that agencies or rivals claim are anticompetitive.

[14]However, smaller values of χ can make it more likely that the game is in states where the leader has a large share, and actually increase expected penalty costs.

Restrictions on Pricing Incentives. Besanko, Doraszelski, and Kryukov (2014) and Besanko, Doraszelski, and Kryukov (2019b) consider the effects of alternative limitations on the dynamic incentives that firms are able to consider using the BDK model, motivated by alternative standards for assessing prices to be predatory.[15] We consider how equilibrium outcomes change when a leader[16]:

- is unable to consider dynamic incentives at all (i.e., the leader has to use an opportunity cost that equals its current production cost).
- is unable to consider AD incentives, but can consider AB incentives, the level of which may change endogenously.

Restrictions on Pricing. An easier-to-implement restriction would prevent the leader from setting a price below its current production cost. Below-cost pricing is often viewed as a necessary, but not sufficient, condition for pricing to be viewed as predatory.[17]

We find equilibria using the homotopy method with an additional set of equations associated with Lagrangian constraints. We find an initial equilibrium using an iterative guess-and-verify approach to identify states where the constraints on the leader's price bind.

Trigger Policies. In practice, political pressure to promote competition may arise only once one firm is established. The social planner also wants to aim for symmetry

[15] While the lack of exit may appear to make the BDKS model a less attractive setting for considering rules motivated by the predation literature, the literature on anticompetitive conduct (e.g., Caves and Porter (1977), Lieberman (1987)) does not assume exit must be possible, and know-how depreciation allows a firm in the BDKS model to try to weaken its rival in a wide range of states.

[16] We have also computed some results imposing these restrictions on both firms, with the no dynamics case corresponding to the static equilibrium considered above. However, allegations of anticompetitive conduct usually focus on the market leader, so focusing on policies that target the leader seems more relevant.

[17] See discussion in the Department of Justice 2008 report "Competition and Monopoly: Single-Firm Conduct Under Section 2 of the Sherman Act", `https://www.justice.gov/sites/default/files/atr/legacy/2009/05/11/236681.pdf`. Of course, application would entail many choices about the appropriate way to measure costs.

once one firm has lowered its costs. We therefore also compute equilibria under "trigger versions" of the policies listed above, meaning that the policy will be introduced as soon as one firm reaches know-how state e' and will then last forever. We assume players know the value of e' when the game begins.

Multiplicity of Equilibria. For the illustrative technology parameters, we find some multiplicity for small τ under the concentration restriction and Leader $p \geq mc$ policies, but the predictions across these equilibria, and the equilibria with no policies, are sufficiently similar that they do not complicate our conclusions. Based on our searches, which do identify multiplicity for other technology parameters, equilibria under the incentive policies are unique.

5.3.2 Predicted Policy Effects.

Figure 9(a)-(d) shows how the policies change HHI^{32}, TS^{PDV} and PS^{PDV} as a function of τ. The only policy to affect outcomes when $\tau = 1$ is the concentration restriction policy as the seller is able to avoid the compliance penalty by not agreeing a price.

Ignoring the slight complications in interpretation caused by multiple equilibria when τ is small, all policies weakly lower expected medium- and long-run concentration for $\tau < 0.8$. However, this reduction is associated with a softening of competition in low know-how states which leads to discounted seller surplus increasing, and a loss in social welfare as production costs are expected to fall more slowly. The policies decrease TS^{PDV} the most when $\tau \approx 0.2$, reflecting the efficiency of the no policy outcome. Comparing across policies, the $p \geq mc$ policy has the smallest effects, reflecting how, with no policies, prices are significantly below production costs in only a relatively small set of states.[18]

For $0.8 \leq \tau \leq 1$, the No Leader AD policy increases TS^{PDV} (and CS^{PDV}, not

[18]Online Appendix D.1 lists prices in a subset of states in the no policy $\tau = 0$ equilibria.

Figure 9: Effects of Policies Introduced at the Start of the Industry's Life on Concentration and Total and Producer Surplus, for the Illustrative Technology Parameters. The compliance costs of the Concentration Restriction policy are not counted as costs to society in total surplus calculations, although they are costs to the sellers.

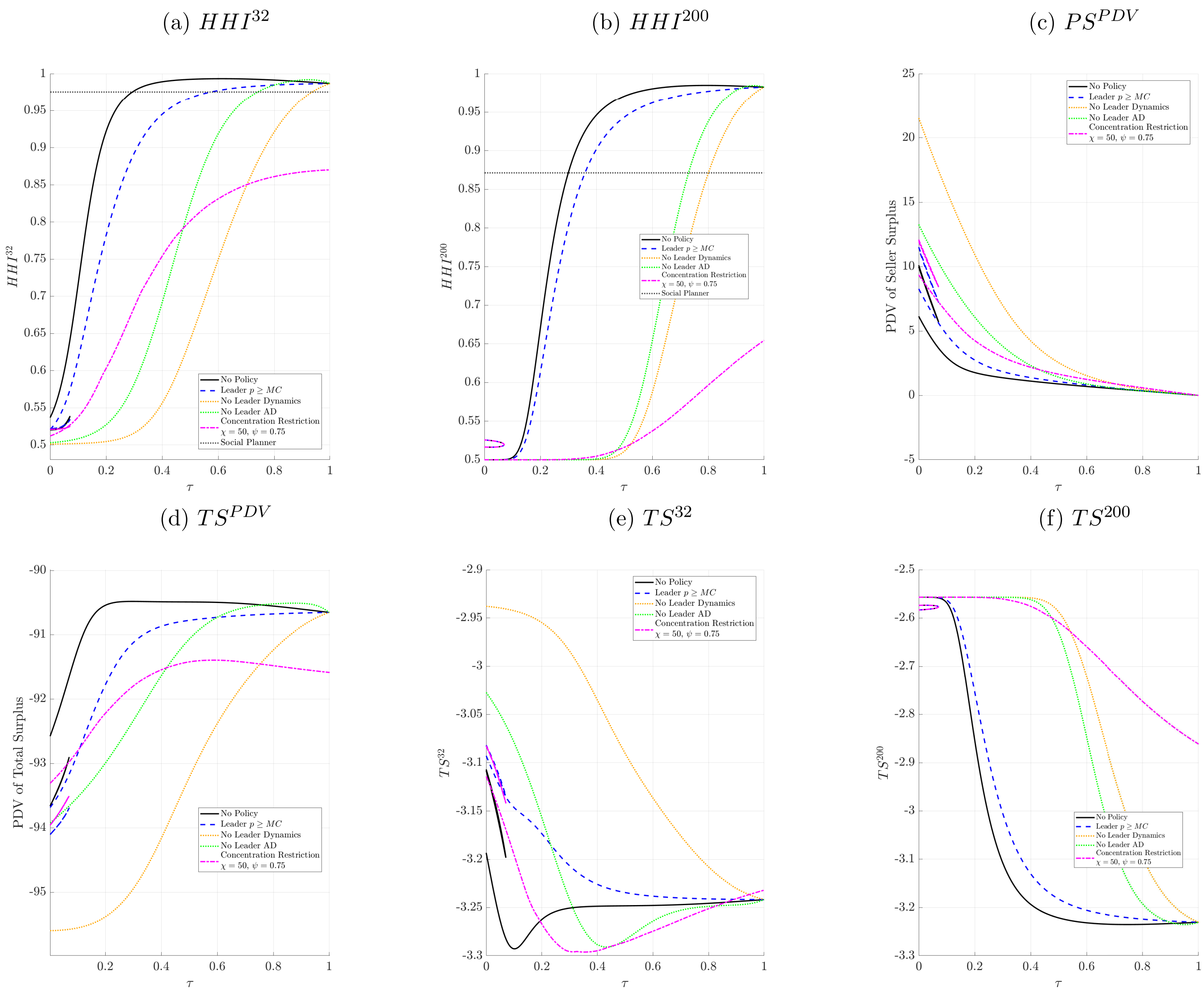
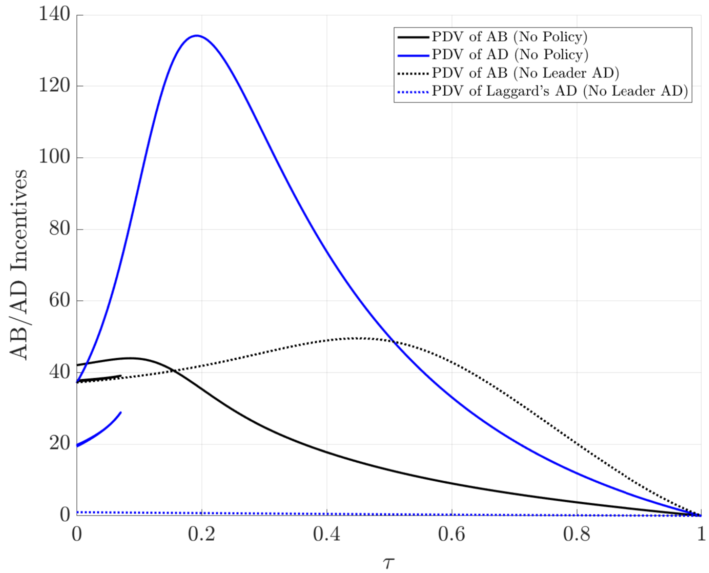


Figure 10: The Present Discounted Value of Sellers' AB and AD Incentives With and Without the No Leader AD Policy for the Illustrative Technology Parameters.

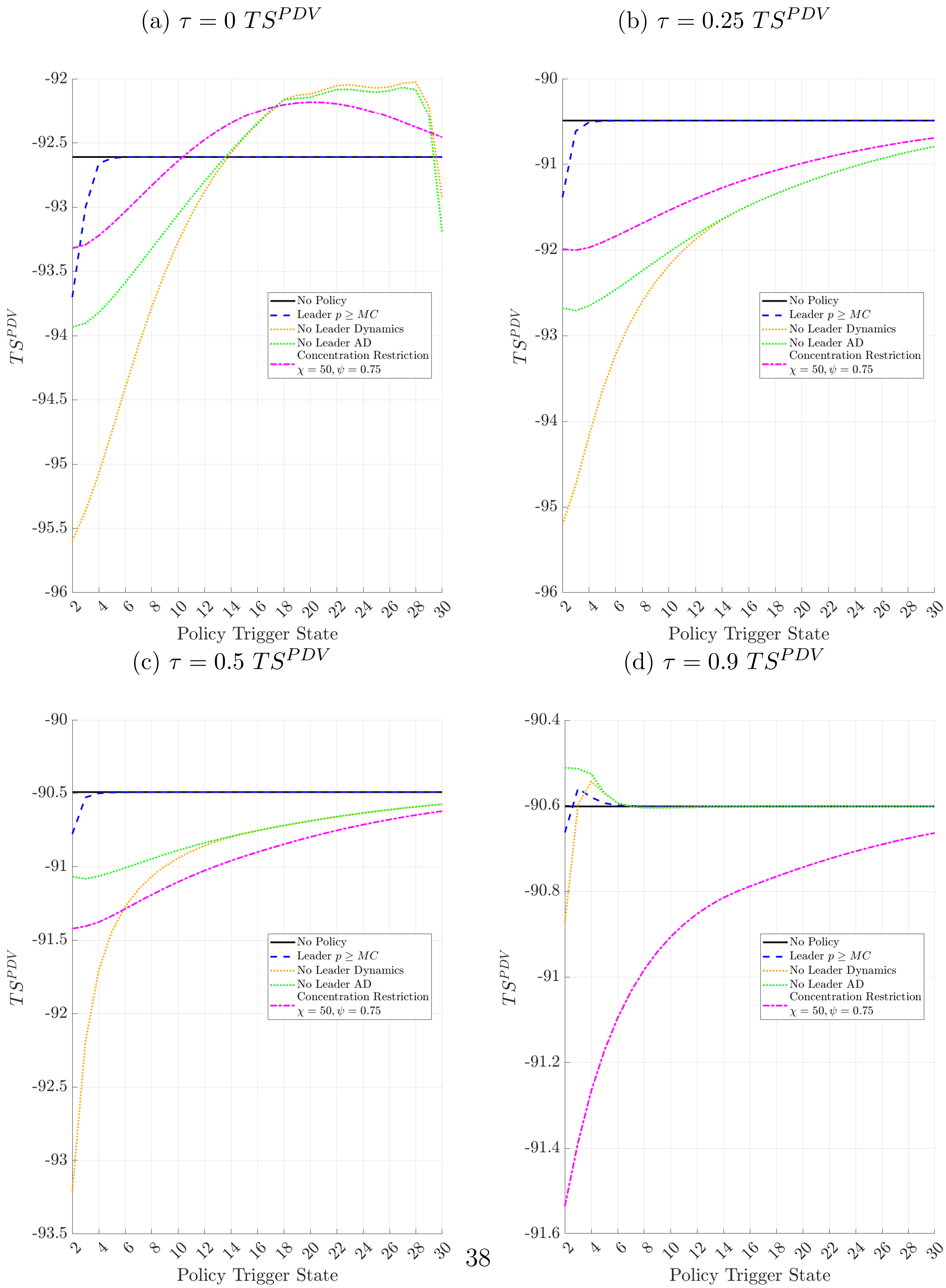


shown), and, for $0.9 \leq \tau < 1$, it also increases HHI^{32}. While applications would rarely assume that $\tau > 0.8$, these changes illustrate the subtle ways that dynamic incentives can interact. Figure 10 shows the PDV of dynamic incentives with no policies, and the value of dynamic incentives that sellers are allowed to consider under the No Leader AD policy. For small and moderate levels of τ, the policy significantly reduces the sum of considered dynamic incentives, but, for $\tau > 0.8$, the sum increases as AB incentives under the policy increase. Further analysis shows that, for $\tau > 0.8$, the No Leader AD policy increases the relative size of the leader's AB incentives in low know-how states, even though buyers have most of the bargaining power, so that the leader moves more quickly down its cost curve.

Figure 9(e) and (f) show that, while the policies lower TS^{PDV} for $\tau < 0.8$, they may increase TS^{32} and TS^{200}, reflecting how they increase the probability that there will be two sellers with low costs. This also suggests that trigger strategies, with appropriate e's, might be able to enhance efficiency.

Figure 11 shows how TS^{PDV} changes for four different τ, as a function of the

Figure 11: Effects of Policies on TS^{PDV} as a Function of the Trigger State for the Illustrative Technology Parameters. The compliance costs of the Concentration Restriction policy are not counted as costs to society in total surplus calculations, although they are costs to sellers.



trigger state e'. As none of the policies directly constrain sellers in state (1,1), the outcomes when the trigger is $e' = 2$ are identical to those when the policy is introduced from the start of the industry. This explains why we observe the No Leader AD policy raising welfare when $\tau = 0.9$ and $e' = 2$.

The figures show that, in fact, the trigger strategies do not raise TS^{PDV} when $\tau = 0.25$ or $\tau = 0.5$. Further analysis shows that TS^{PDV} is reduced for τs even closer to zero: for example, the incentive (concentration restriction) policies with triggers of $e' = 20$ lower TS^{PDV} if $\tau > 0.06$ (0.08), although the reductions in welfare are small if $\tau > 0.8$. Therefore, though equilibrium medium- or long-run concentration is too high with no policies for $\tau = 0.25$ or $\tau = 0.5$, none of the considered pro-competition policies raise efficiency.

The incentive and concentration restriction policies are, however, predicted to raise TS^{PDV} when $\tau = 0$ and e' is chosen so that the policy only applies once one firm has reached, or almost reached, the bottom of its cost-curve (recall $m = 15$). The increase in efficiency may seem surprising given that concentration is too low when $\tau = 0$ and there are no policies, but this is, in fact, exactly the reason why efficiency increases. The laggard anticipates that the policy will help it catch-up once the leader reaches e'. Therefore, the laggard's incentives to compete before this state is reached are reduced, and the leader moves down its cost-curve more quickly, increasing concentration and surplus. This can lower medium-run surplus and raise medium-run concentration: for example, TS^{32} is lower with the policies, than with no policy, if $e' \geq 13$ and HHI^{32} increases if $e' \geq 20$.

6 Conclusion

The dynamic competition literature has assumed that sellers compete by setting prices or quantities. The existing applied literature on bargaining has focused on settings where either market structure is fixed or prices and market shares do not have direct effect on firms' future competitiveness. We extend the dynamic competition and applied bargaining literatures by embedding buyer-seller bargaining into an existing and tractable model of dynamic price competition, with the standard assumption that sellers make take-it-or-leave-it price offers nested as a special case. By doing so, our model allows the allocation of bargaining power to affect market structure, and endogenous market structure to affect sellers' outside options, their dynamic incentives and negotiated outcomes. These interactions are likely to be important in the types of capital goods industries where negotiations are common and which have motivated much of the policy-oriented dynamic competition literature in international trade and industrial organization.

We find that, for a wide-range of technology parameters, the mark-ups that leaders charge when they set prices causes leads to be relatively short-lived, so that sellers' dynamic incentives are relatively weak and, because sellers move down their cost curves relatively slowly, production costs are quite high. These patterns can change quickly when buyers have even quite limited bargaining power, as mark-ups are reduced and longer lasting leads incentivize sellers to try to get ahead. As a result, conclusions that would be drawn assuming sellers set prices may not generalize even to cases where sellers have most, but not quite all, of the bargaining power.

We show how the allocation of bargaining power affect the design of optimal subsidy policies and the effects of policies that might be expected to promote competition. For example, subsidies that would be optimal with seller price-setting only require small deviations from this assumption to be welfare-reducing. Of course, it is an empirical question how bargaining power is actually allocated between buyers and sellers, and our results suggest that the accurate estimation of bargaining power in models with dynamic price competition is an important direction for future research. While our framework naturally extends to considering forward-looking buyers (Sweeting, Jia, Hui, and Yao (2022)), a convincing empirical analysis would likely need to include additional real-world features, such as multi-unit and multi-period contracts, that the current paper has abstracted away from.

References

ABBRING, J. H., AND J. R. CAMPBELL (2010): "Last-In First-Out Oligopoly Dynamics," *Econometrica*, 78(5), 1491–1527.

ABBRING, J. H., J. R. CAMPBELL, J. TILLY, AND N. YANG (2018): "Very Simple Markov-Perfect Industry Dynamics: Theory," *Econometrica*, 86(2), 721–735.

AN, Y., AND W. ZHAO (2019): "Dynamic Efficiencies of the 1997 Boeing-McDonnell Douglas Merger," *RAND Journal of Economics*, 50(3), 666–694.

ASKER, J., C. FERSHTMAN, J. JEON, AND A. PAKES (2020): "A Computational Framework for Analyzing Dynamic Auctions: The Market Impact of Information Sharing," *RAND Journal of Economics*, 51(3), 805–839.

BARWICK, P. J., M. KALOUPTSIDI, AND N. B. ZAHUR (2019): "China's Industrial Policy: An Empirical Evaluation," Discussion Paper 26075, National Bureau of Economic Research.

BENKARD, C. L. (2000): "Learning and Forgetting: The Dynamics of Aircraft Production," *American Economic Review*, 90(4), 1034–1054.

——— (2004): "A Dynamic Analysis of the Market for Wide-Bodied Commercial Aircraft," *Review of Economic Studies*, 71(3), 581–611.

BESANKO, D., U. DORASZELSKI, AND Y. KRYUKOV (2014): "The Economics of Predation: What Drives Pricing When There is Learning-by-Doing?," *American Economic Review*, 104(3), 868–97.

——— (2019a): "How Efficient is Dynamic Competition? The Case of Price as Investment," *American Economic Review*, 109(9), 3339–64.

——— (2019b): "Sacrifice Tests for Predation in a Dynamic Pricing Model: Ordover and Willig (1981) and Cabral and Riordan (1997) meet Ericson and Pakes (1995)," *International Journal of Industrial Organization*, 70, 102522.

Besanko, D., U. Doraszelski, Y. Kryukov, and M. Satterthwaite (2010): "Learning-by-Doing, Organizational Forgetting, and Industry Dynamics," *Econometrica*, 78(2), 453–508.

Cabral, L. M., and M. H. Riordan (1994): "The Learning Curve, Market Dominance, and Predatory Pricing," *Econometrica*, 62(5), 1115–1115.

Caves, R. E., and M. E. Porter (1977): "From Entry Barriers to Mobility Barriers: Conjectural Decisions and Contrived Deterrence to New Competition," *Quarterly Journal of Economics*, 91(2), 241–261.

Collard-Wexler, A., G. Gowrisankaran, and R. S. Lee (2019): ""Nash-in-Nash" Bargaining: A Microfoundation for Applied Work," *Journal of Political Economy*, 127(1), 163–195.

Crawford, G. S., R. S. Lee, M. D. Whinston, and A. Yurukoglu (2018): "The Welfare Effects of Vertical Integration in Multichannel Television markets," *Econometrica*, 86(3), 891–954.

Dasgupta, P., and J. Stiglitz (1988): "Learning-By-Doing, Market Structure and Industrial and Trade Policies," *Oxford Economic Papers*, 40(2), 246–268.

Dorn, J. (2023): "Dynamic Bargaining Between Hospitals and Insurers," Discussion paper, Princeton University.

Draganska, M., D. Klapper, and S. B. Villas-Boas (2010): "A Larger Slice or a Larger Pie? An Empirical Investigation of Bargaining Power in the Distribution Channel," *Marketing Science*, 29(1), 57–74.

Ericson, R., and A. Pakes (1995): "Markov-Perfect Industry Dynamics: A Framework for Empirical Work," *Review of Economic Studies*, 62(1), 53–82.

Fudenberg, D., and J. Tirole (1983): "Learning-by-Doing and Market Performance," *Bell Journal of Economics*, 14(2), 522–530.

Ghemawat, P. (1985): "Building Strategy on the Experience Cirve," *Harvard Business Review*, 63(2), 143–149.

Gowrisankaran, G., A. Nevo, and R. Town (2015): "Mergers When Prices Are Negotiated: Evidence from the Hospital Industry," *American Economic Review*, 105(1), 172–203.

Grennan, M. (2013): "Price Discrimination and Bargaining: Empirical Evidence from Medical Devices," *American Economic Review*, 103(1), 145–177.

Ho, K., and R. S. Lee (2017): "Insurer Competition in Health Care Markets," *Econometrica*, 85(2), 379–417.

Horn, H., and A. Wolinsky (1988): "Bilateral Monopolies and Incentives for Merger," *RAND Journal of Economics*, 19(3), 408–419.

Kalai, E. (1977): "Proportional Solutions to Bargaining Situations: Interpersonal Utility Comparisons," *Econometrica*, 45(7), 1623–1630.

Kalouptsidi, M. (2018): "Detection and Impact of Industrial Subsidies: The Case of Chinese Shipbuilding," *The Review of Economic Studies*, 85(2), 1111–1158.

Kim, M. (2014): "Strategic Responses to Used Goods Markets: Airbus and Boeing," Discussion paper, University of Oklahoma.

Leahy, D., and J. P. Neary (1999): "Learning by Doing, Precommitment and Infant-Industry Promotion," *Review of Economic Studies*, 66(2), 447–474.

Lee, R. S., and K. Fong (2013): "Markov Perfect Network Formation: An Applied Framework for Bilateral Oligopoly and Bargaining in Buyer-Seller Networks," Discussion paper, New York University.

Lieberman, M. B. (1987): "Postentry Investment and Market Structure in the Chemical Processing Industries," *RAND Journal of Economics*, 18(4), 533–549.

Maskin, E., and J. Tirole (2001): "Markov Perfect Equilibrium: I. Observable Actions," *Journal of Economic Theory*, 100(2), 191–219.

Neary, J. P., and D. Leahy (2000): "Strategic Trade and Industrial Policy Towards Dynamic Oligopolies," *Economic Journal*, 110(463), 484–508.

Pakes, A., and P. McGuire (1994): "Computing Markov-Perfect Nash Equilibria: Numerical Implications of a Dynamic Differentiated Product Model," *RAND Journal of Economics*, 25(4), 555–589.

Sweeting, A., D. Jia, S. Hui, and X. Yao (2022): "Dynamic Price Competition, Learning-by-Doing, and Strategic Buyers," *American Economic Review*, 112(4), 1311–33.

Sweeting, A., J. W. Roberts, and C. Gedge (2020): "A Model of Dynamic Limit Pricing with an Application to the Airline Industry," *Journal of Political Economy*, 128(3), 1148–1193.

Sweeting, A., X. Tao, and X. Yao (2023): "Dynamic Games with Asymmetric Information: Implications for Mergers," Discussion paper, University of Maryland.

Tiew, A. (2024): "Flailing Firms and Joint Operating Agreements: An Application to US Local Daily Print Newspapers from 1932 to 1992," Discussion paper, Harvard University.

Yang, C. (2020): "Vertical Structure and Innovation: A Study of the SoC and Smartphone Industries," *RAND Journal of Economics*, 51(3), 739–785.

ONLINE APPENDICES FOR "Bargaining and Dynamic Competition" by Deng, Jia, Leccese and Sweeting

April 2024

A Analytical Proofs

A.1 Proofs of Proposition 1.

Recall Proposition 1.

Proposition 1 *In a model with any $m \leq M$,*

1. *if $\tau = 1$, equilibrium prices will equal marginal production costs in all states, for all ρ and δ.*

2. *there will be a unique symmetric MPNE when*

   (a) *$\delta = 0$ for all ρ and τ, or*

   (b) *$\tau = 1$ for all ρ and δ.*

Proof of Part 1. The structure of the proof is to show that VS must be zero in every state, which implies that price will equal marginal production costs. From text equation (9), $\tau = 1$ implies that all markups Φ are equal to zero. Equation (11) therefore implies that $\mathbf{VS_1} = 0$, and equation (10) implies $\widehat{\mathbf{c_1}} = \mathbf{c_1}$, so that equation (9) now implies $p_1^*(\mathbf{e}) = c_1(\mathbf{e})$ in all states.

Proof of Part 2(a). Follows from the recursive proof of Besanko, Doraszelski, Kryukov, and Satterthwaite (2010), and the fact logit demand implies there can only be one (static) MPNE in the absorbing state (M, M).

Proof of Part 2(b). Uniqueness of prices follows immediately from the proof of part 1, and the choice probabilities of a static seller are unique given prices.

A.2 Proof of Proposition 2.

Proposition 2 *For $m = M = 2$ and $\delta = 0$, the unique symmetric equilibrium will have the following properties*

1. *equilibrium $D_1^*(1,2)(\tau) < \frac{1}{2}$ for all τ.*
2. *equilibrium $D_1^*(1,2)(\tau)$ is strictly decreasing in τ.*
3. *for $t \geq 2$, HHI^t is strictly increasing in τ.*
4. *there exists a τ^* such that $TS^{PDV}(\tau^*) = TS^{SP}$, $TS^{PDV}(\tau)$ is strictly increasing in τ for $\tau \in (0, \tau^*)$ and strictly decreasing in τ for $\tau \in (\tau^*, 1)$.*

A.2.1 Characterization

Given $M = 2$ and $\delta = 0$, the equilibrium is fully characterized by the equilibrium condition for $D_1^*(1,2)$, which can be simplified to

$$H(x,\tau) - H\left(\frac{1}{2}, \tau\right) = c(1) - c(2), \tag{18}$$

where $H(x)$ is defined as

$$H(x,\tau) := \log \frac{1-x}{x} - \frac{1-\beta+\beta x}{1-\beta}\left[\widetilde{\Phi}\left(x, \frac{\tau}{1-\tau}\right) - (1-\beta)\widetilde{\Phi}\left(1-x, \frac{\tau}{1-\tau}\right)\right] \text{ for } x, \tau \in [0,1],$$

where $\widetilde{\Phi}(x,z)$ is defined as

$$\widetilde{\Phi}(x,z) := \frac{\log \frac{1}{1-x}}{zx + (1-x)\log \frac{1}{1-x}} \text{ for } x \in [0,1] \text{ and } z \in [0,\infty),$$ [19]

[19] $\widetilde{\Phi}(x,\infty)$ is defined to be 0 for all $x \in [0,1]$.

and is a reformulated version of the mark-up condition $\Phi(x,\tau)$ where the second argument in $\widetilde{\Phi}(x,z)$ is $\frac{\tau}{1-\tau}$.

As negotiated prices are a transfer from buyers to sellers, we can express expected total surplus in any state as a function of the choice probabilities and firm costs only, i.e., $\sum_{k=1,2} D_k(\mathbf{e})\left(\log \frac{1}{D_k(\mathbf{e})} - c_k(\mathbf{e})\right)$ which does not depend on τ. In the $M=2$ and $\delta=0$ case, TS^{PDV}, for a game starting at (1,1), can be written as the following function of $D_1(1,2)$,

$$TS^{PDV}(x) = \frac{\beta}{1-\beta+\beta x}\left[x\log\frac{1}{x} + (1-x)\log\frac{1}{1-x} - (c(1)-c(2))x - \log 2\right] + \frac{\log 2 - \beta c(2) - (1-\beta)c(1)}{1-\beta}.$$

We define D_1^{SP} as the choice probability that the social planner would choose in state (1,2). The social planner, would, of course, use choice probabilities of $\frac{1}{2}$ in states (1,1) and (2,2) where the suppliers are symmetric, with the firm with the highest ε making the sale.

When $\delta=0$ and $x=D_1(1,2)$,

$$HHI^t = (x^2+(1-x)^2)(1-x)^{t-1} + \frac{1}{2}(1-(1-x)^{t-1})$$

for $t \geq 2$ as the state will either be (1,2) with concentration $(x^2+(1-x)^2)$ or (2,2) with concentration $\frac{1}{2}$.

A.2.2 Preliminary Results.

Lemma A.1 *$H(x,\tau)$ is strictly decreasing in $x \in (0,1)$.*

Proof of Lemma A.1. Since there is a unique equilibrium when $\delta=0$ for any parameterization of $c(e)$, $H(x,\tau)$ must be strictly monotone in $x \in (0,1)$. Otherwise, there will be multiple equilibria—i.e., multiple solutions to (18)—for some value of $c(1)-c(2)$.

It suffices to show that $H(x,\tau)$ is decreasing on some interval in $(0,1)$. When $\tau = 1$, it is easy to see that $H(x,\tau)$ is strictly decreasing $x \in (0,1)$. When $\tau < 1$, we have that $\widetilde{\Phi}\left(x, \frac{\tau}{1-\tau}\right)$ strictly increases with $x \in (0,1)$. Therefore, $H(x,\tau)$ decreases with x when $\widetilde{\Phi}\left(x, \frac{\tau}{1-\tau}\right) > (1-\beta)\widetilde{\Phi}\left(1-x, \frac{\tau}{1-\tau}\right)$. This completes the proof. ■

Lemma A.2 *$TS^{PDV}(x)$ is strictly increasing in $x \in (0, D_1^{SP})$ and strictly decreasing in $x \in (D_1^{SP}, 1)$, where $D_1^{SP} \in (0, 1/2)$ solves*

$$\log\frac{1}{x} - \frac{1}{1-\beta}\log\frac{1}{1-x} = c(1) - c(2) - \frac{\beta}{1-\beta}\log 2. \tag{19}$$

Proof of Lemma A.2. The proof immediately follows from the fact that

$$\frac{dTS^{PDV}(x)}{dx} = \frac{\beta(1-\beta)}{(1-\beta+\beta x)^2}\left\{\log\frac{1}{x} - \frac{1}{1-\beta}\log\frac{1}{1-x} - \left[c(1) - c(2) - \frac{\beta}{1-\beta}\log 2\right]\right\}.$$

■

A.2.3 Proofs of Proposition 2

Proof of Proposition 2.1: equilibrium $D_1^*(1,2)(\tau) < \frac{1}{2}$ for all τ. By Lemma A.1, the left-hand side of (18) is strictly decreasing in x. It is clear that the left-hand side equals 0 when $x = \frac{1}{2}$. Since the right-hand side is strictly positive, the solution must be less than $\frac{1}{2}$. ■

Proof of Proposition 2.2: equilibrium $D_1^*(1,2)(\tau)$ is strictly decreasing in τ. Applying the implicit function theorem to (18) yields that

$$\frac{\partial x}{\partial \tau} = -\frac{H_\tau(x,\tau) - H_\tau\left(\frac{1}{2},\tau\right)}{H_x(x,\tau)}.$$

By Lemma A.1, $H_x(x,\tau) < 0$, and by Proposition 2.1, $x < \frac{1}{2}$. So to complete the proof, it suffices to show that $H_\tau(x,\tau) < H_\tau\left(\frac{1}{2},\tau\right)$ for $x \in \left(0, \frac{1}{2}\right)$.

Note that

$$H_\tau(x,\tau) = -\frac{1-\beta+\beta x}{1-\beta}\left[\widetilde{\Phi}_z(x,z) - (1-\beta)\widetilde{\Phi}_z(1-x,z)\right]\frac{dz}{d\tau}, \text{ with } z = \frac{\tau}{1-\tau}.$$

Therefore,

$$\begin{aligned} & H_\tau(x,\tau) < H_\tau\left(\frac{1}{2},\tau\right) \\ \iff & -\frac{1-\beta+\beta x}{1-\beta}\left[\widetilde{\Phi}_z(x,z) - (1-\beta)\widetilde{\Phi}_z(1-x,z)\right] < -\frac{(2-\beta)\beta}{2(1-\beta)}\widetilde{\Phi}_z\left(\frac{1}{2},z\right) \\ \iff & -\left[\widetilde{\Phi}_z(x,z) - (1-\beta)\widetilde{\Phi}_z(1-x,z)\right] < -\frac{(1-\beta)(2-\beta)\beta}{2(1-\beta)(1-\beta+\beta x)}\widetilde{\Phi}_z\left(\frac{1}{2},z\right). \end{aligned} \tag{20}$$

In fact,

$$\widetilde{\Phi}_z(x,z) = -\frac{x\log\frac{1}{1-x}}{\left[zx + (1-x)\log\frac{1}{1-x}\right]^2}, \text{ and } \widetilde{\Phi}_z\left(\frac{1}{2},z\right) = -\frac{2\log 2}{(z+\log 2)^2} < 0.$$

So the right-hand side of (20) is decreasing in $x \in \left(0, \frac{1}{2}\right)$.

Next, we show that the left-hand side of (20) increases with $x \in \left(0, \frac{1}{2}\right)$. It suffices to show that $-\widetilde{\Phi}_z(x,z)$ increases with $x \in \left(0, \frac{1}{2}\right)$. In fact,

$$\begin{aligned} -\widetilde{\Phi}_{zx}(x,z) &= \frac{\left(\log\frac{1}{1-x} + \frac{x}{1-x}\right)\left[zx + (1-x)\log\frac{1}{1-x}\right] - 2x\log\frac{1}{1-x}\left(z + 1 - \log\frac{1}{1-x}\right)}{\left[zx + (1-x)\log\frac{1}{1-x}\right]^3} \\ &= \frac{zx\left(\frac{x}{1-x} - \log\frac{1}{1-x}\right) + \log\frac{1}{1-x}\left[(1+x)\log\frac{1}{1-x} - x\right]}{\left[zx + (1-x)\log\frac{1}{1-x}\right]^3}. \end{aligned}$$

It is easy to verify that $\frac{x}{1-x} > \log\frac{1}{1-x} > \frac{x}{1+x}$ for $x \in \left(0, \frac{1}{2}\right)$. As a result, $-\widetilde{\Phi}_{zx}(x,z) > 0$.

Combining the facts that the right-hand side of (20) is decreasing in $x \in \left(0, \frac{1}{2}\right)$, the left-hand side of (20) is increasing in $x \in \left(0, \frac{1}{2}\right)$, and both sides are equal at $x = \frac{1}{2}$, we can conclude that (20) holds for $x \in \left(0, \frac{1}{2}\right)$. This completes the proof. ∎

Proof of Proposition 2.3: equilibrium expected concentration (HHI^t) in any period $t \geq 2$, strictly increases in τ. The derivative of HHI^t with respect to x is, for $t \geq 2$

$$-\frac{(1-2x)(1-x)^{t-2}(3+t-2x-2tx)}{2}$$

which is negative for any $x < \frac{1}{2}$. As equilibrium $D_1^*(1,2) < \frac{1}{2}$ (Proposition 2.1) and equilibrium $D_1^*(1,2)$ decreases in τ (Proposition 2.2), HHI^t increases in τ. ■

Proof of Proposition 2.4 : there exists a τ^* such that $TS^{PDV}(\tau^*) = TS^{SP}$, and $TS^{PDV}(\tau^*)$ is increasing in τ for $\tau \in (0, \tau^*)$ and decreasing in τ for $\tau \in (\tau^*, 1)$.

As TS^{PDV} only depends on $D_1^*(1,2)$, $TS^{PDV}(\tau^*) = TS^{SP}$ if $D_1^*(1,2)(\tau^*) = D_1^{SP}$. Because $D_1^*(1,2)(\tau)$ decreases with τ (Proposition 2.2), and Lemma A.2, the result follows if

$$D_1^*(1,2)(\tau = 0) > D_1^{SP} > D_1^*(1,2)(\tau = 1).$$

By (18) and the monotonicity of $H(x,\tau)$ in x (Lemma A.1), the above inequality is equivalent to

$$H(D_1^{SP},0) - H\left(\frac{1}{2},0\right) - [c(1)-c(2)] > 0 > H(D_1^{SP},1) - H\left(\frac{1}{2},1\right) - [c(1)-c(2)]. \quad (21)$$

We first show that $0 > H(D_1^{SP}, 1) - H\left(\frac{1}{2}, 1\right) - [c(1) - c(2)]$. In fact,

$$H(D_1^{SP},1) - H\left(\frac{1}{2},1\right) - [c(1)-c(2)] = \frac{\beta}{1-\beta}\left(\log\frac{1}{1-D_1^{SP}} - \log 2\right) < 0,$$

where the equality is due to that D_1^{SP} solves (19), and the inequality is due to that $D_1^{SP} < \frac{1}{2}$ (Lemma A.2).

It only remains to show that $H(D_1^{SP}, 0) - H\left(\frac{1}{2}, 0\right) - [c(1) - c(2)] > 0$. Note that

$$
\begin{aligned}
& H(D_1^{SP}, 0) - H\left(\frac{1}{2}, 0\right) - [c(1) - c(2)] \\
&= \log\left(\frac{1 - D_1^{SP}}{D_1^{SP}}\right) - [c(1) - c(2)] - \left[\frac{1}{(1-\beta)(1 - D_1^{SP})} - \frac{1-\beta}{D_1^{SP}} - \frac{\beta(2-\beta)}{1-\beta}\right] - H\left(\frac{1}{2}, 0\right).
\end{aligned}
$$

Again, substituting (19) into the right-hand side of the above equation yields that

$$
H(D_1^{SP}, 0) - H\left(\frac{1}{2}, 0\right) - [c(1) - c(2)] = L(D_1^{SP}) - L\left(\frac{1}{2}\right),
$$

where

$$
L(x) := \frac{\beta}{1-\beta} \log \frac{1}{1-x} - \left[\frac{1}{(1-\beta)(1-x)} - \frac{1-\beta}{x}\right].
$$

Since

$$
L'(x) = -\frac{1 - \beta(1-x)}{(1-\beta)(1-x)^2} - \frac{1-\beta}{x^2} < 0,
$$

$L(x)$ is decreasing in x, and $L(D_1^{SP}) > L\left(\frac{1}{2}\right)$. Therefore

$$
H(D_1^{SP}, 0) - H\left(\frac{1}{2}, 0\right) - [c(1) - c(2)] = L(D_1^{SP}) - L\left(\frac{1}{2}\right) > 0.
$$

■

B Numerical Methods for Finding Equilibria

B.1 Homotopies.

This Appendix provides details of our implementation of the homotopy algorithm using the example of how we use a sequence of homotopies to try to enumerate the number of equilibria that exist for different values of (ρ, δ) for given values of τ. Our implementation of other homotopies, for example, by varying τ, is similar to a single step in this sequence. We describe the procedure assuming that $M = 30$ and that we are using the price and value formulation of the equations. Identical procedures apply using the choice probability formulation, except that it is necessary to use the choice probabilities to calculate prices and values in order to check whether the identified solutions are different enough to be labeled as distinct equilibria.

B.1.1 Preliminaries

We identify equilibria at particular gridpoints in (ρ, δ) space. We specify a 201-point evenly-spaced grid for the forgetting rate $\delta \in [0, 0.2]$ and a 41-point evenly-spaced grid for the learning progress ratio $\rho \in [0.6, 1]$. The state space of the game is defined by a (30×30) grid of values of the know-how of each firm.

B.1.2 System of Equations Defining Equilibrium

An MPNE is defined by a system of equations (one VS^* equation (text equation (17)) for each of 900 states and one p^* equation (text equation (6)) for each of 900 states. The grouping of all of these equations is denoted F.

B.1.3 Homotopy Algorithm: Overview

The idea of the homotopy is to trace out an equilibrium correspondence as one of the parameters of interest is changed, holding the others fixed. Starting from any equilibrium, the numerical algorithm traces a path where a parameter (such as δ), and the vectors $VS^*(\mathbf{e})$ and $p^*(\mathbf{e})$ are changed together so that the equations F continue to hold, by solving a system of differential equations. The differential equation solver does not return equilibria exactly at the gridpoints so it is necessary to interpolate between the solutions returned by the solver. Homotopies can be run starting from different equilibria and varying different parameters. When these different homotopies return interpolated solutions at the same gridpoint it is necessary to define a numerical rule for when two different solutions should be counted as different equilibria.

B.1.4 Procedure Details

Step 1: Finding Equilibria for $\delta = 0$. The first step is to find an equilibrium (i.e., a solution to the 1,800 equations) for $\delta = 0$ for each value of ρ on the grid. There will be a unique MPNE for $\delta = 0$, as, in this case, movements through the state space are unidirectional, so that the state will eventually end up in the state (M, M) where no more learning is possible.

We solve for an equilibrium using the Levenberg-Marquardt algorithm implemented using `fsolve` in MATLAB, where we supply analytic gradients for each equation. The solution for the previous value of ρ are used as starting values. To ensure that the solutions are precise, we use a tolerance of 10^{-7} for the sum of squared values of each equation, and a relative tolerance of 10^{-14} for the price and value variables that we are solving for.

Step 2: δ-Homotopies. Using the notation of Besanko, Doraszelski, Kryukov, and Satterthwaite (2010), we explore the correspondence

$$F^{-1}(\rho) = \{(\mathbf{V}^*, \mathbf{p}^*, \delta) | F(\mathbf{V}^*, \mathbf{p}^*; \rho, \delta) = \mathbf{0}, \quad \delta \in [0, 1]\},$$

The homotopy approach follows the correspondence as a parameter, s, changes (in our analysis, s could be δ, ρ or τ). Denoting $\mathbf{x} = (\mathbf{V}^*, \mathbf{p}^*)$, $F(\mathbf{x}(s), \delta(s), \rho) = \mathbf{0}$

can be implicitly differentiated to find how $\mathbf{x}$ and δ must change for the equations to continue to hold as s changes.

$$\frac{\partial F(\mathbf{x}(s), \delta(s), \rho)}{\partial \mathbf{x}}\mathbf{x}'(s) + \frac{\partial F(\mathbf{x}(s), \delta(s), \rho)}{\partial \delta}\delta'(s) = \mathbf{0}$$

where $\frac{\partial F(\mathbf{x}(s),\delta(s),\rho)}{\partial \mathbf{x}}$ is a (1,800 x 1,800) matrix, $\mathbf{x}'(s)$ and $\frac{\partial F(\mathbf{x}(s),\delta(s),\rho)}{\partial \delta}$ are both (1,800 x 1) vectors and $\delta'(s)$ is a scalar. The solution to these differential equations will have the following form, where $y_i'(s)$ is the derivative of the ith element of $\mathbf{y}(s) = (\mathbf{x}(s), \delta(s))$,

$$y_i'(s) = (-1)^{i+1} \det\left(\left(\frac{\partial F(\mathbf{y}(s), \rho)}{\partial \mathbf{y}}\right)_{-i}\right)$$

where $_{-i}$ means that the ith column is removed from the (1,801 x 1,801) matrix $\frac{\partial F(\mathbf{y}(s),\rho)}{\partial \mathbf{y}}$.

To implement the path-following procedure, we use the FORTRAN routine FIXPNS from HOMPACK90, with the ADIFOR 2.0D automatic differentiation package used to evaluate the sparse Jacobian $\frac{\partial F(\mathbf{y}(s),\rho)}{\partial \mathbf{y}}$ and the STEPNS routine is used to find the next point on the path.[20,21]

The FIXPNS routine will return solutions at values of δ that are not equal to the gridpoints. Therefore we adjust the code so that after *each* step, the algorithm checks whether a gridpoint has been passed and, if so, the routine ROOTNX is used to calculate the equilibrium at the gridpoint, using information on the solutions at either side.[22]

The time taken to run a homotopy is usually between one hour and seven hours,

[20]STEPNS is a predictor-corrector algorithm where hermetic cubic interpolation is used to guess the next point, and an iterative procedure is then used to return to the path.

[21]For details of the HOMPACK subroutines, please consult manual of the algorithm at `https://users.wpi.edu/~walker/Papers/hompack90,ACM-TOMS_23,1997,514-549.pdf`.

[22]It can happen that the ROOTNX routine stops prematurely so that the returned solution is not exactly at the gridpoint value of δ. We do not use the small proportion of solutions where the difference is more than 10^{-6}. Varying this threshold does not affect the reported results. We also need to decide whether the equations have been solved accurately enough so that the values and strategies can be treated as equilibria. The criteria that we use is that solutions where the value of each equation residual should be less than 10^{-10}. Otherwise, the solution is rejected. In practice, the rejected solutions typically have residuals that are much larger than 10^{-10}.

when it is run on UMD's BSWIFT cluster (a moderately sized cluster for the School of Behavioral and Social Sciences).

Step 3: Enumerating Equilibria. Once we have collected the solutions at each of the (ρ, δ) gridpoints we need to identify which solutions represent distinct equilibria, taking into account that small differences may arise because of numerical differences that are within our tolerances. For this paper, we use the rule that solutions count as different equilibria if at least one element of the price vector differs by more than 0.001.

Step 4: ρ-Homotopies. With a set of equilibria from the δ-homotopies in hand, we can perform the next round of our criss-crossing procedure which alternates ρ-homotopies and δ-homotopies, which we run in both directions (e.g., decreasing ρ as well as increasing ρ). We use equilibria found in the last round as starting points.[23]

This second round of homotopies can also help us to deal with gridpoints where the first round δ-homotopies identify no equilibria because a homotopy run stops (or takes a long sequence of infinitesimally small steps). As noted by Besanko, Doraszelski, Kryukov, and Satterthwaite (2010) (p. 467), the homotopies may stop if they reach a point where the evaluated Jacobian $\frac{\partial F(\mathbf{y}(s),\rho)}{\partial \mathbf{y}}$ has less than full rank. Suppose, for example, that the δ-homotopy for $\rho = 0.8$ stops at $\delta = 0.1$, so we have no equilibria for δ values above 0.1. Homotopies that are run from gridpoints where we did find equilibria with higher values of δ and higher or lower values of ρ may fill in some of the missing equilibria.

Step 5: Repeat Steps 3, 2 and 4 to Identify Additional Equilibria Using New Equilibria as Starting Points. We use the procedures described in Step 3

[23]In practice, using all new equilibria could be computationally prohibitive. We therefore use an algorithm that continues to add new groups of 10,000 starting points when we find that using additional starting points yields a significant number of equilibria that have not been identified before. We have experimented with different rules, and have found that alternative algorithms do not find noticeably more equilibria, across the parameter space, than the algorithm that we use.

to identify new equilibria at the gridpoints. These new equilibria are used to start new sets of δ-homotopies, which in turn can identify equilibria that can be used for new sets of ρ-homotopies. This iterative process is continued until the number of additional equilibria that are identified in a round has no noticeable effect on the heatmaps which show the number of equilibria. For the Besanko, Doraszelski, Kryukov, and Satterthwaite (2010), $\tau = 0$ case, this happens after 8 rounds.

B.2 Method for Finding Equilibria Based on Three Reformulated Equations in the $M = 3$ Model.

We now describe the alternative method that we use to identify equilibria when $M = 3$.

As described in the text, the equilibrium conditions can be reformulated in terms of the probability that seller 1 is chosen in each state. If we restrict ourselves to symmetric equilibria then, together with the restriction that $D_1(e_1, e_2) = 1 - D_1(e_2, e_1)$, then there are just three unknown probabilities. We will use $D_1(1, 2)$, $D_1(1, 3)$ and $D_1(2, 3)$. The equilibrium equations for these three states are:

$$\sigma \log \left(\frac{1}{D_1^*(e_1, e_2)} - 1 \right) - p_1^*(e_1, e_2) + p_2^*(e_1, e_2) = 0, \tag{22}$$

and, from text Section 3.2,

$$\mathbf{p_1} = \mathbf{\Phi}(\mathbf{D_1}) + \mathbf{c_1} - \beta(\mathbf{Q_1} - \mathbf{Q_2})(\mathbf{I} - \beta\mathbf{Q_2})^{-1}[\mathbf{D_1} \circ \mathbf{\Phi}(\mathbf{D_1})]. \tag{23}$$

in vector form, so that we can substitute prices to express the equations (22) in terms of choice probabilities only.

We proceed in the following steps for a given (ρ, δ, τ) combination.

Step 1. Define a grid of possible values for $D_1(1, 2)$ and $D_1(1, 3)$. For each, we use a vector [1e-10, 1e-9, 1e-7, 1e-6, 1e-5, (0.0001:(0.9999-0.0001)/200:0.9999), 1-1e-5, 1-1e-6, 1-1e-7, 1-1e-8, 1-1e-9, 1-1e-10].

Step 2. For every combination on the grid, find for the value of $D_1(2,3)$ which solves the equilibrium equation for state (2,3), and record the values of the equations (22) for states (1,2) and (1,3), in matrices $M(1,2)$ and $M(1,3)$.[24]

Step 3. Use MATLAB `contour` command to define the shapes where the $M(1,2)$ and $M(1,3)$ surfaces are equal to zero.

Step 4. Count all of the intersections of these curves, using the user-defined MATLAB function `InterX` command.[25]

Of course, the contours are calculated using interpolation so the solutions are therefore not quite exact. Therefore,

Step 5. Using the solutions from the contour intersections as starting points, solve the equilibrium equations using `fsolve`.

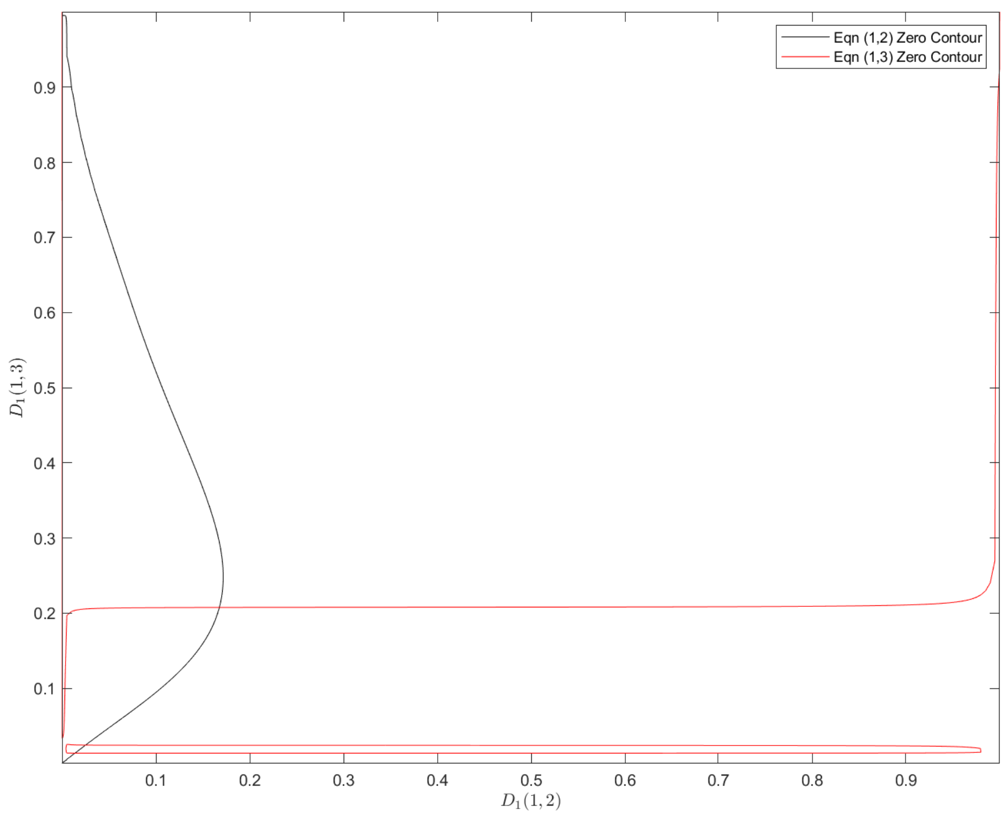
Step 6. Count the number of solutions where at least one choice probability is different from all of the other equilibria by at least 5e-4.

To give a sense of the procedure, consider the parameters $\rho = 0.1$, $\delta = 0.05$ and $\tau = 0.0$. Figure B.2 shows the contour plot. The three intersections between the black and red lines in the bottom left of the figure identify equilibria.

[24] We have not been able to prove uniqueness, but all of the the examples we have looked at there is a unique solution.

[25] `https://www.mathworks.com/matlabcentral/fileexchange/22441-curve-intersections`.

Figure B.2: Illustration of the Contour Plot for $\rho = 0.1$, $\delta = 0.05$ and $\tau = 0$.



C Additional Analysis for the $M = 3$ Model

This Appendix provides some additional analysis for the $M = 3$ model, including for results that are mentioned briefly in the text.

C.1 Equilibrium Strategies and Outcomes for Polar Cases for $\rho = 0.3$ and $\delta = 0.03$.

We use $\rho = 0.3$ and $\delta = 0.03$ as our example parameters in the $M = 3$ model. Table C.2 shows equilibrium prices, sale probabilities and welfare outcomes for (i) the social planner solution, (ii) the equilibrium when $\tau = 0$, and (iii) the equilibrium when $\tau = 1$. The table also shows the probabilities that the industry is in each state after 4 periods (state (3,3) cannot be reached) and 32 periods.

C.2 TS^{PDV}, HHI^{32} and Dynamic Incentives for Alternative ρ and δ.

Our $M = 3$ analysis in the text uses $\rho = 0.3$ and $\delta = 0.03$ as an illustration. We observe that TS^{PDV}, HHI^{32} and the value of dynamic incentives increase as τ increases from zero, with TS^{PDV} and the value of dynamic incentives displaying inverted-U relationships with τ, and concentration a monotonically increasing relationship. The non-monotonic path of dynamic incentives reflects how leads tend to be short-lived when $\tau = 0$, but last longer, so that the value of attaining a lead can increase, as τ rises even though the seller's share of surplus is falling. The non-monotonic path of TS^{PDV} reflects how the social planner would choose more concentration than produced by the $\tau = 0$ equilibrium but less than the equilibrium with $\tau = 1$.

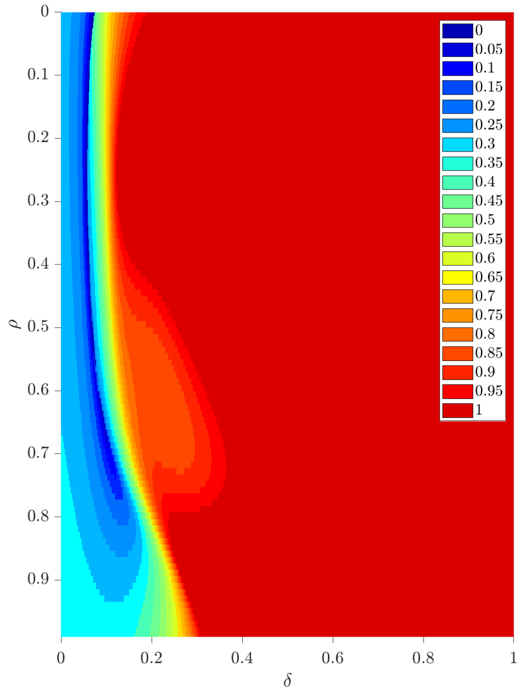
Figure C.2(a)-(c) show the values of τ that maximize TS^{PDV}, HHI^{32} and the PDVs of seller dynamic incentives for all possible ρ and δ combinations using 0.05 steps of τ from 0 to 1. We use the maximum value of the outcome for the small set of (ρ, δ, τ) parameters with multiple equilibria. This choice does affect the value of τ that maximizes the statistic for some (ρ, δ) combinations (see Appendix C.3 for an

Table C.2: $M = 3$ with $\rho = 0.3$ and $\delta = 0.03$: Strategies and Outcomes in Polar Cases. In these tables, firm 2 is assumed to be the leader.

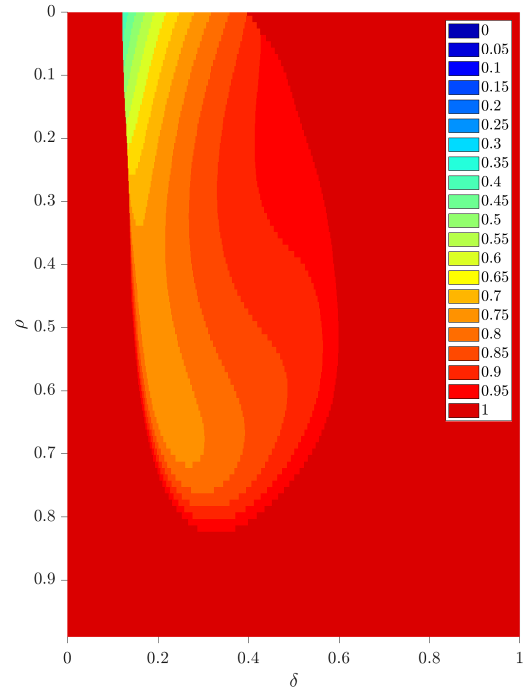
	(a) Social Planner			(b) $\tau = 0$ Equilibrium			(c) $\tau = 1$ Equilibrium		
	Laggard State Firm 1			Laggard State Firm 1			Laggard State Firm 1		
e_1	1	2	3	1	2	3	1	2	3
c_1	10	3	1.483	10	3	1.483	10	3	1.483
Δ	0.03	0.059	0.087	0.03	0.059	0.087	0.03	0.059	0.087
Prices and Laggard Sale Probability									
$e_2 = 1$	$p_1 = 10$ $p_2 = 10$ $D_1 = 0.5$			$p_1 = 3.378$ $p_2 = 3.378$ $D_1 = 0.5$			$p_1 = 10$ $p_2 = 10$ $D_1 = 0.5$		
$e_2 = 2$	$p_1 = 10$ $p_2 = 3$ $D_1 = 0.084$	$p_1 = 3$ $p_2 = 3$ $D_1 = 0.5$		$p_1 = 4.355$ $p_2 = 3.274$ $D_1 = 0.253$	$p_1 = 2.918$ $p_2 = 2.918$ $D_1 = 0.500$		$p_1 = 10$ $p_2 = 3$ $D_1 = 0.001$	$p_1 = 3$ $p_2 = 3$ $D_1 = 0.500$	
$e_2 = 3$	$p_1 = 10$ $p_2 = 1.483$ $D_1 = 0.114$	$p_1 = 3$ $p_2 = 1.483$ $D_1 = 0.58$	$p_1 = 1.483$ $p_2 = 1.483$ $D_1 = 0.5$	$p_1 = 4.344$ $p_2 = 3.395$ $D_1 = 0.279$	$p_1 = 3.059$ $p_2 = 3.225$ $D_1 = 0.542$	$p_1 = 3.339$ $p_2 = 3.339$ $D_1 = 0.5$	$p_1 = 10$ $p_2 = 1.483$ $D_1 = 0.000$	$p_1 = 3$ $p_2 = 1.483$ $D_1 = 0.180$	$p_1 = 1.483$ $p_2 = 1.483$ $D_1 = 0.5$
4 Period Probability Distribution									
$e_2 = 1$	4.57E-05			8.18E-05			2.72E-05		
$e_2 = 2$	0.0070	0.0190		0.0087	0.050		0.0060	0.0001	
$e_2 = 3$	0.8233	0.1506	0	0.5741	0.3670	0	0.9930	0.0009	0
32 Period State Probability Distribution									
$e_2 = 1$	4.30-09			7.32E-09			1.05E-12		
$e_2 = 2$	2.63E-05	0.0012		1.57E-05	0.00086		6.28E-07	1.97E-05	0
$e_2 = 3$	0.0642	0.1458	0.7888	0.0159	0.1556	0.8277	0.9955	0.0019	0.0026
Expected Welfare Outcomes									
	TS	**CS**	**PS**	**TS**	**CS**	**PS**	**TS**	**CS**	**PS**
PDV	-38.342	-	-	-38.742	-56.169	17.427	-40.702	-40.702	0
4 period	-2.051	-	-	-2.622	-2.797	0.174	-1.492	-1.492	0
32 period	-1.006	-	-	-0.959	-2.621	1.661	-1.481	-1.481	0

Figure C.2: Panels (a)-(c): Values of τ Maximizing TS^{PDV}, HHI^{32} and the PDV of Seller Dynamic Incentives in an $M = 3$ Model. For values of (ρ, δ) with multiple equilibria we use the equilibrium that maximizes the value of the statistic. Panel (d) shows equilibrium, for various τ, and social planner HHI^{32} as a function of δ when $\rho = 0.3$.

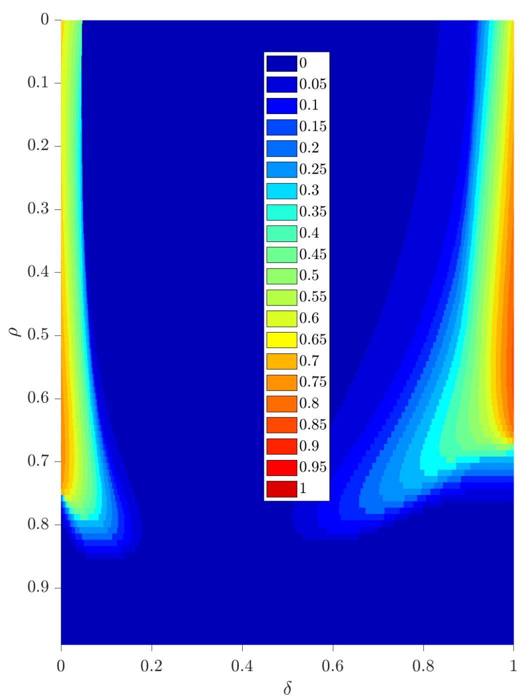
(a) TS^{PDV}



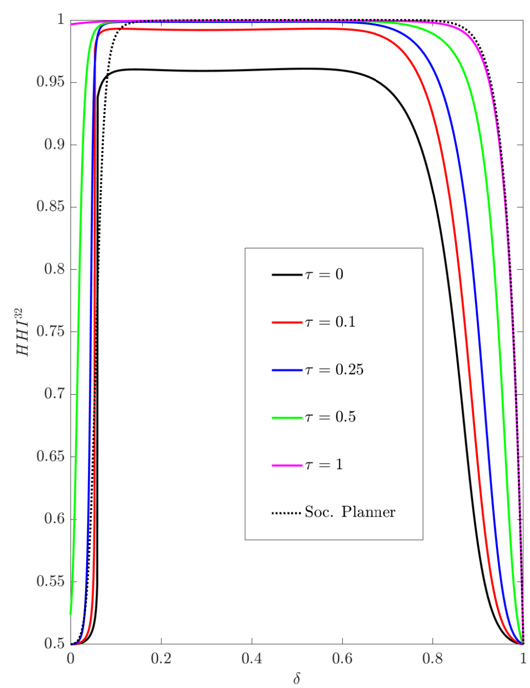
(b) HHI^{32}



(c) PDV of AB+AD Incentives



(d) HHI^{32}



example).

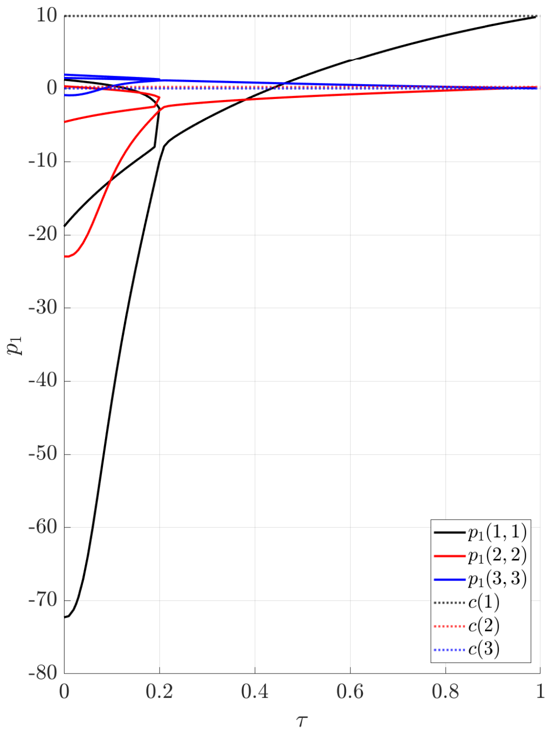
The pattern that raising τ increases concentration is fairly general, and in the cases where HHI is maximized for $\tau < 1$, the level of concentration when $\tau \approx 1$ is exceptionally high. However, the non-monotonicity of TS^{PDV} and dynamic incentives is only a systematic pattern when δ is small and, in the case of dynamic incentives, LBD effects are also at least moderately important.

The patterns change when δ is larger. Panel (d) shows what happens to HHI^{32} in equilibrium, for various τ, and for the social planner, as δ varies and $\rho = 0.3$. For $\delta > 0.1$, the social planner solution involves a single firm making almost every sale, until $\delta > 0.8$, at which point forgetting is so likely that the firm making a sale is unlikely to lower its costs. Equilibria when $0.1 \leq \delta \leq 0.8$ also lead to high concentration, although slightly less than the social optimum due to the leader charging a markup. For $\delta \geq 0.1$, welfare tends to be maximized when $\tau = 1$, consistent with $\tau = 1$ maximizing the probability that a firm with the lowest possible cost will make the sale.

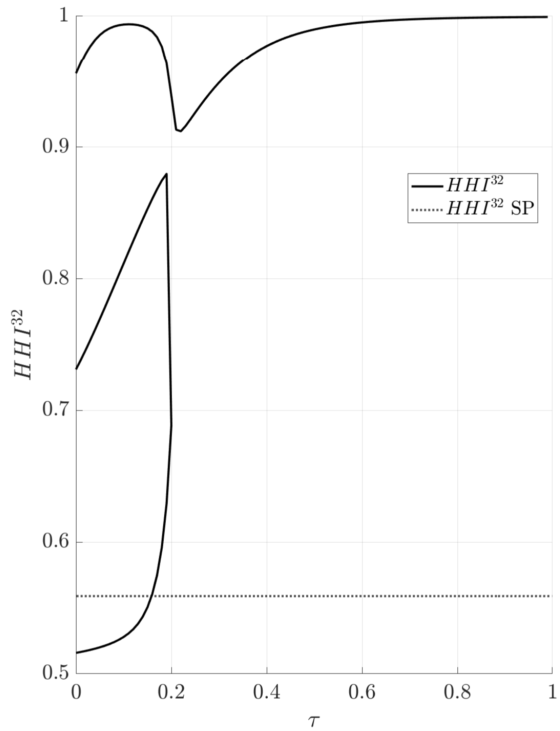
For a wide-range of parameters, dynamic incentives are maximized when $\tau = 0$. This reflects how concentration is high, and leads tend to last a long time, even when $\tau = 0$ for $\delta > 0.1$. When leads last a long time, the lead lengthening effect cannot increase dynamic incentives significantly, and is dominated by how increasing τ shrinks sellers' future profits. When ρ is high, competition is always fairly symmetric, and increasing τ does not cause leads to lengthen very much so that the lead-lengthening effect is also small.[26]

Figure C.3: Prices in Symmetric States, HHI^{32}, TS^{PDV} and Dynamic Incentives along τ-homotopy paths when $\rho = 0.02$ and $\delta = 0.058$.

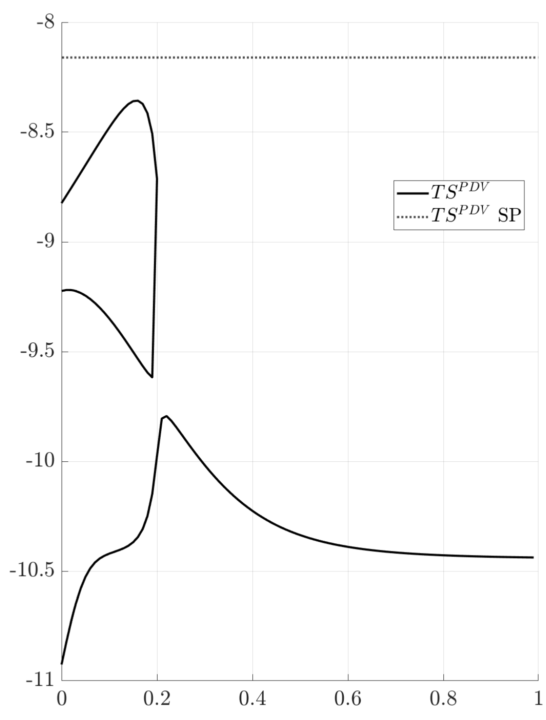
(a) Prices in Symmetric States



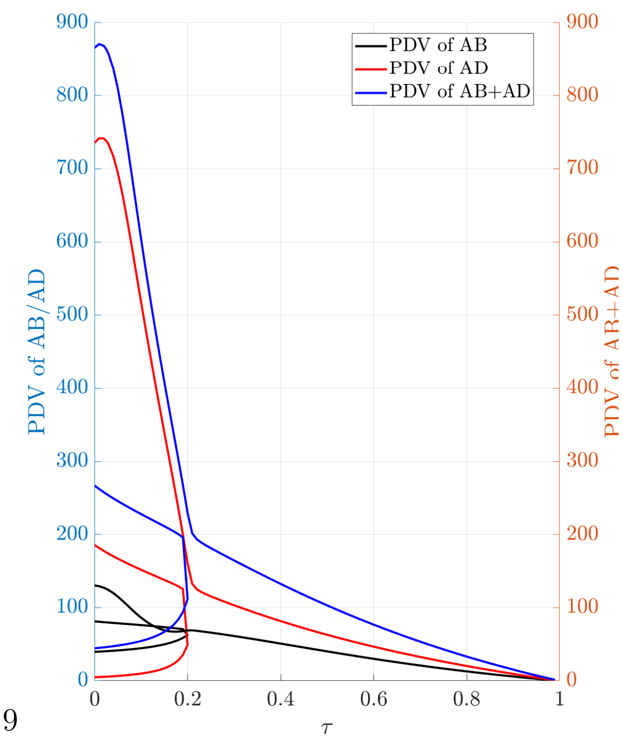
(b) HHI^{32}



(c) TS^{PDV}



(d) Dynamic Incentives



C.3 Analysis for $\rho = 0.02$ and $\delta = 0.058$: Parameters with Multiple Equilibria.

$\rho = 0.3$ and $\delta = 0.03$ support a single equilibrium for all τ. In this subsection, we show how changing τ changes outcomes and dynamic incentives for $\rho = 0.02$ and $\delta = 0.058$. For these parameters, multiplicity exists for $\tau \leq 0.202$. $\rho = 0.02$ implies that production costs fall from 10 to 0.2 when know-how increases from state 1 to 2, i.e., learning-by-doing effects are extreme and sellers in states 2 and 3 have almost the same costs, but a state 2 seller is at risk of experiencing a very large cost increase (and cost disadvantage) if it does not make a sale. As δ is not too large, the social planner would prefer to have both firms in states 2 and 3, with $D_1^{SP}(2,3) \approx 0.8$ so that both firms are very likely to be low cost in the next period.

Figure C.3 shows the values of equilibrium HHI^{32}, TS^{PDV}, prices in symmetric states, and the PDVs of AB and AD incentives along τ-homotopy paths that start from the three equilibria that are identified when $\tau = 0$. The $\tau = 0$ equilibrium that is on the path that continues all of the way to $\tau = 1$ has a pronounced diagonal trench in the sense of having very low, and below cost, "aggressive" prices in all symmetric states, high equilibrium concentration ($D_1^*(1,2) \approx 0.07$), and large AD incentives. In contrast, prices in asymmetric states are higher (for example, in (3,1) prices are 6.1 and 10.5), reflecting how a laggard has limited incentive to try to catch up when symmetric competition is so fierce. This equilibrium minimizes total surplus as the costs associated with reduced variety are larger than the benefits of lower expected costs. The two other $\tau = 0$ equilibria involve firms setting higher prices in symmetric states, including above cost prices in state (3,3). In one of the equilibria concentration is lower than the social planner would choose.

As τ increases, the benefits of achieving a lead in the diagonal trench equilibrium decrease, so that there is a range of τ, between 0.1 and 0.22, where equilibrium

[26] For example if $\rho = 0.9$ and $\delta = 0.03$, increasing τ from 0 to 0.2, causes the lead of a firm in (2,1) to last an expected 2.4, rather than 2.3, periods. The PDV of AD incentives does increase slightly over this range, but this is offset by the value of AB incentives falling by slightly more.

concentration falls, and symmetric prices rise. On the other hand, concentration increases on the loop from the other two $\tau = 0$ equilibria. TS^{PDV} is maximized on the loop path that does not extend past $\tau = 0.202$.

For the $M = 30$ model with illustrative technology parameters ($\rho = 0.75, \delta = 0.023$), we also find three equilibria when $\tau = 0$. Two of them have diagonal trenches, and they are on a loop that does not extend past $\tau = 0.07$. Based on that example, we have been asked by discussants and seminar participants whether diagonal trench equilibria are always on paths that do not continue once τ is large enough, as this would suggest one might be able to view variation in buyer bargaining power as some type of equilibrium selection device. This $M = 3$ example provides a counter-example to this conjecture, and, while diagonal trench equilibria are often eliminated, we have identified similar counterexamples for $M = 30$ as well.

C.4 Introducing an Outside Good and Varying σ.

We follow Besanko, Doraszelski, Kryukov, and Satterthwaite (2010) in assuming that there is no outside good and that σ, which controls the degree of product differentiation, equals 1. In this appendix we examine how far our conclusions about the effects of bargaining power on outcomes depend on these assumptions, assuming $\rho = 0.3$ and $\delta = 0.03$.

C.4.1 Outside Good.

When the buyer is able to choose not to purchase, sellers face additional competition which will constrain markups. One can construct intuitions where this either weakens the incentive of a firm to establish a lead over its rival (as it will reduce the return to establishing a lead) or strengthens it (as competition from the outside good may make it even harder for a laggard to catch up).

We introduce an outside good by assuming that, in every period, buyers have a third option, with indirect utility $v - p_0 + \varepsilon_0$, where p_0 is an exogenous parameter that we can use to control the attractiveness of the outside good. Besanko, Doraszelski, and Kryukov (2014), who consider a model where only a single seller may be active, allow an outside good with a baseline $p_0 = 10$.

Figure C.4(a) shows the equilibrium values of HHI^{32} for $\tau = 0$, 0.2 and 0.5 as a function of p_0. Figure C.4(c) and (e) show the PDV of AB and AD incentives. The BDKS model can be viewed as the limiting case where $p_0 \to \infty$ (i.e., we extend the right-hand edge of the figure further to the right). The effects of the outside good are small unless $p_0 < 7$, in which case dynamic incentives increase and concentration (reflecting the dominance of one seller over its rival) increases slightly. However, for $p_0 \approx 10$, all measures are very similar to those when an outside good is assumed not to exist, and introducing an outside good has little effect on the comparisons between different τs.

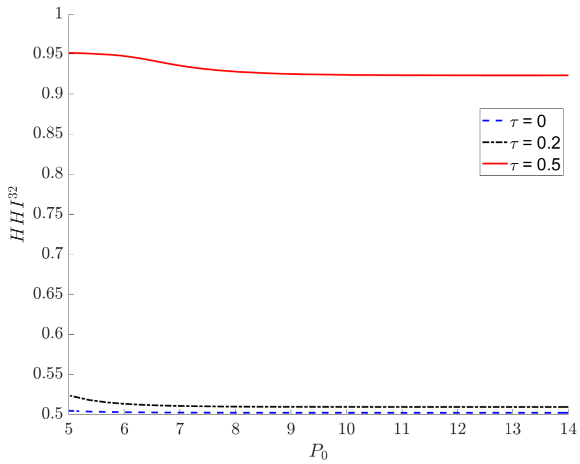
C.4.2 Changing Product Differentiation.

As σ increases, it becomes more likely that, in any period, the buyer will have a strong preference for one of the sellers, so that seller competition is softened. This will tend to make it more likely that purchases will be evenly split across sellers, although the expectation of future sales and softened competition could increase a firm's incentive to lower its own costs.

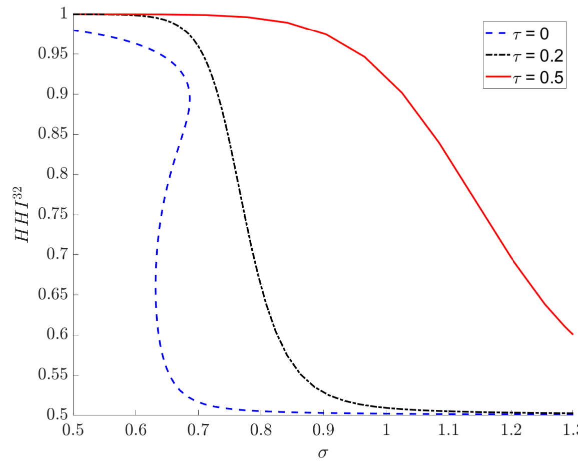
Figure C.4 shows the same statistics as σ is varied, assuming that there is no outside good. Increasing σ from 1 lowers concentration, but does not dramatically change how giving buyers bargaining power affects outcomes. On the other hand, reducing σ to 0.8, or lower, causes equilibrium concentration to increase sharply and can introduce multiple equilibria when $\tau = 0$, illustrated by the $\tau = 0$ path bending back on itself. Higher concentration is associated with leads lasting longer when $\tau = 0$, which tends to lead to dynamic incentives monotonically declining in τ.

Figure C.4: HHI^{32} and the PDV of Dynamic Incentives as a Function of the Exogenous Price of an Outside Good (panels a, c, and e) and Product Differentiation (b, d and f) when $M = 3$, $\rho = 0.3$ and $\delta = 0.03$.

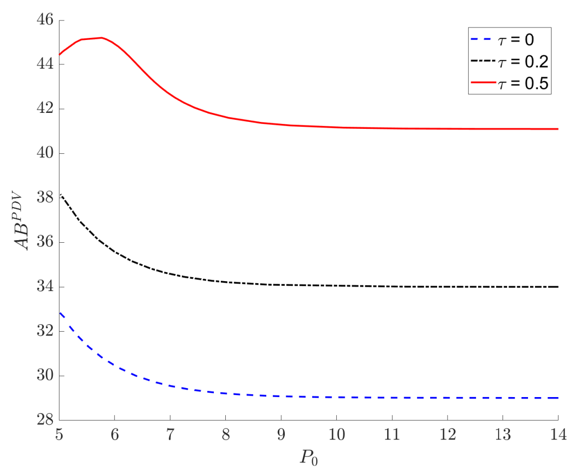
(a) Outside Good: HHI^{32}



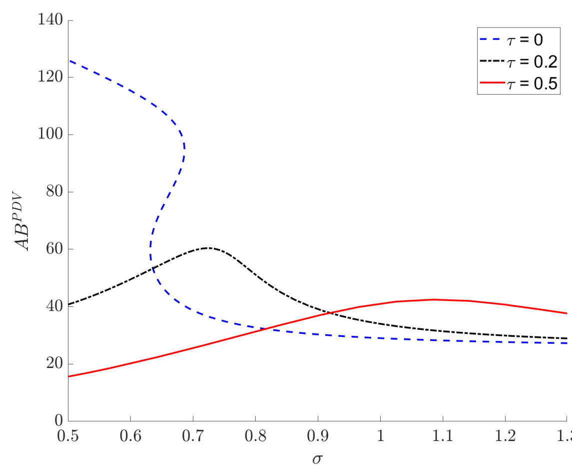
(b) Differentiation: HHI^{32}



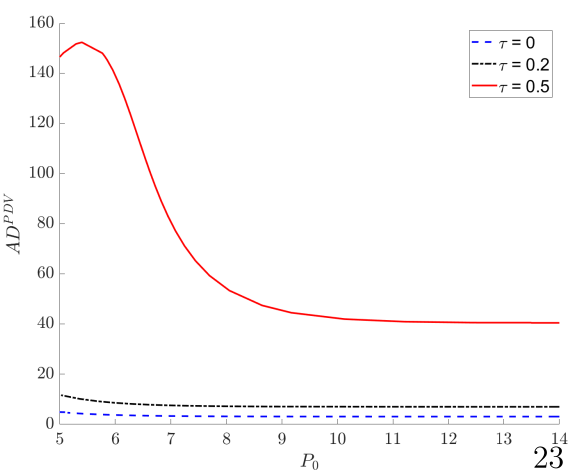
(c) Outside Good: PDV of AB Incentives



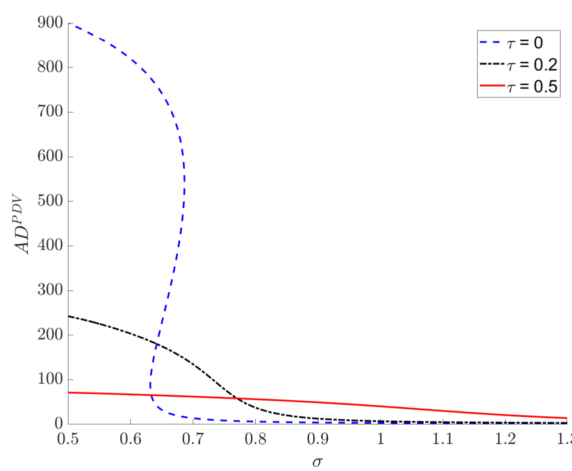
(d) Differentiation: PDV of AB Incentives



(e) Outside Good: PDV of AD Incentives



(f) Differentiation: PDV of AD Incentives



C.5 Subsidies.

We calculate subsidies that could implement the social planner outcome. The text shows that the subsidies that would be optimal if $\rho = 0.3$, $\delta = 0.03$ and $\tau = 0$ lower welfare, relative to the no subsidy equilibrium, if $\tau \geq 0.06$.

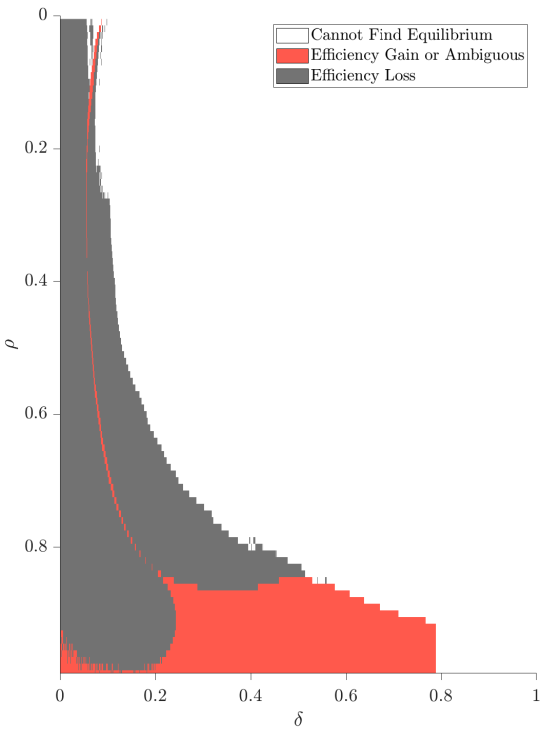
We try to investigate whether optimal $\tau = 0$ subsidies lower welfare for other technologies when $\tau > 0$. We specifically consider $\tau = 0.2$ so that sellers have most, but not all, of the bargaining power and an economist might assume that a $\tau = 0$ analysis would provide a reasonable approximation if they do not appreciate how quickly strategies and outcomes change when τ increases from zero.

Figure C.5(a) shows that $\tau = 0$ subsidies lower welfare if $\tau = 0.2$ for a wide range of parameters as long as LBD effects are not too limited ($\rho < 0.85$). The red areas indicate cases where there is either an efficiency gain or the existence of multiple equilibria when $\tau = 0.2$ means that we cannot sign whether welfare is increased or reduced. The white area indicates parameters where we cannot solve for $\tau = 0$ subsidies or we cannot solve for the equilibrium when $\tau = 0.2$ when these subsidies are in place.

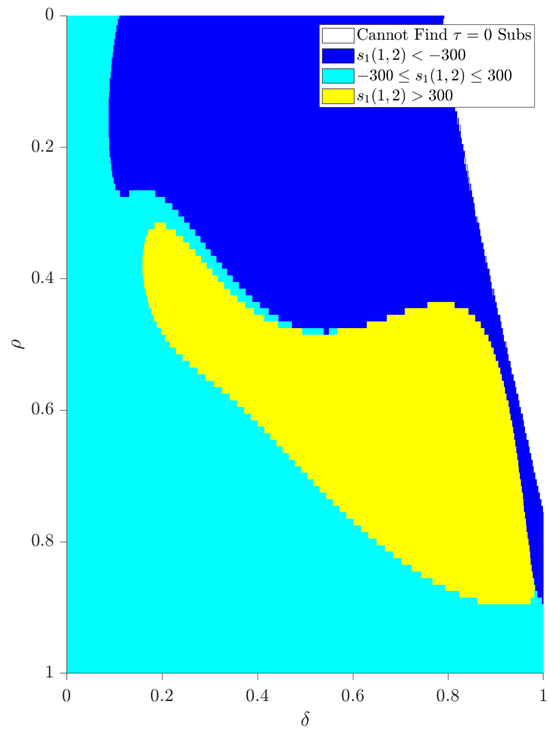
The fact that the white areas cover so many parameters may seem surprising. It reflects the extreme values of some of the required subsidies or the extreme values of the equilibrium choice probabilities when these subsidies are in place. As illustration, panel (b) shows the level of the subsidy the planner would want to give to a laggard making a sale in state (1,2). For $\delta > 0.1$, the subsidies or taxes can be extremely large. It is also interesting how a small change in ρ can switch the optimal scheme from providing a laggard with a very large subsidy to requiring the laggard to pay a very large tax.

Figure C.5: Optimal $\tau = 0$ Subsidies and Welfare in the $M = 3$ Model. Panel (a) shows, for the values of ρ and δ for which we can identify equilibria, parameters where $\tau = 0$ optimal subsidies increase or decrease welfare when $\tau = 0.2$ (compared to no subsidy equilibria). Panel (b) shows the value of $\tau = 0$ subsidies to the laggard in state (1,2). A negative subsidy is a tax on a laggard sale.

(a) Welfare Effect of $\tau = 0$ Subsidies if $\tau = 0.2$



(b) Level of $\tau = 0$ Laggard Subsidy in State (1,2)



D Additional Analysis for the $M = 30$ Model

D.1 Equilibria for $\tau = 0$, $\rho = 0.75$ and $\delta = 0.023$.

Table D.1 lists strategies in a subset of states for the three equilibria that exist for the illustrative technology parameters when $\tau = 0$. All equilibria have negative prices in the initial state (1,1). The B and C equilibria are characterized by a "diagonal trench" with lower prices when firms are symmetric or almost symmetric. For example, prices in (10,10) are 2.43, whereas prices in (29,10), where costs are weakly lower, are 5.39 and 5.15. The trench in equilibrium B does not extend to the highest levels of know-how, and it has slightly lower $HHI^{1,000}$, as it is more likely that the sellers will be symmetric in the long-run. Equilibrium A, with the lowest $HHI^{1,000}$, has prices that vary less with know-how once both firms reached know-how states 3 or 4. However, the leader sets lower prices when the laggard is in know-how states 1 or 2, so that, at the start of the game, it is more likely that one of the sellers will move down its cost curve more quickly, so that HHI^{32} is higher.

D.2 Concentration and Bargaining Power.

Text section 5 shows, for the illustrative technology parameters, that:

- concentration is similar in dynamic equilibria and in an equilibrium where firms price statically, and concentration is lower than the social planner would choose, when $\tau = 0$.
- static and dynamic equilibria are identical when $\tau = 1$, and concentration is above the level that the social planner would choose.
- with static pricing, concentration increases fairly steadily as τ increases from 0 to 1, whereas, with dynamic behavior, concentration increases sharply (and actually overshoots its $\tau = 1$ level) as τ increases from zero.

Table D.1: Equilibria in the Besanko, Doraszelski, Kryukov, and Satterthwaite (2010) Model for Illustrative Parameters ($\delta = 0.023$, $\rho = 0.75$) and $\tau = 0$ in the $M = 30$ and $m = 15$ Model.

						Eqm. A $HHI^{32} = 0.537$ $HHI^{1,000} = 0.500$		Eqm. B $HHI^{32} = 0.520$ $HHI^{1,000} = 0.516$		Eqm. C $HHI^{32} = 0.520$ $HHI^{1,000} = 0.527$	
e_1	e_2	c_1	c_2	Δ_1	Δ_2	p_1	p_2	p_1	p_2	p_1	p_2
1	1	10.00	10.00	0.0230	0.0230	-0.54	-0.54	-1.63	-1.63	-1.61	-1.61
2	1	7.50	10.00	0.0455	0.0230	4.91	7.21	5.16	7.60	5.15	7.60
2	2	7.50	7.50	0.0455	0.0455	4.22	4.22	0.77	0.77	0.77	0.77
3	1	6.34	10.00	0.0674	0.0230	5.82	8.18	6.56	8.71	6.55	8.70
3	2	6.34	7.50	0.0674	0.0455	4.65	5.46	4.06	5.97	4.05	5.97
3	3	6.34	6.34	0.0674	0.0674	5.11	5.11	1.49	1.49	1.47	1.47
4	1	5.62	10.00	0.0889	0.0230	5.95	8.29	6.67	8.55	6.67	8.54
4	2	5.62	7.50	0.0889	0.0455	4.86	5.85	5.46	7.08	5.46	7.08
4	3	5.62	6.34	0.0889	0.0674	5.08	5.42	3.81	5.53	3.80	5.54
4	4	5.62	5.62	0.0889	0.0889	5.22	5.22	1.72	1.72	1.70	1.70
10	1	3.85	10.00	0.2076	0.0230	5.89	8.16	6.14	7.71	6.13	7.69
10	2	3.85	7.50	0.2076	0.0455	5.05	6.06	5.75	6.43	5.75	6.42
10	3	3.85	6.34	0.2076	0.0674	5.20	5.81	5.80	6.31	5.80	6.31
10	8	3.85	4.22	0.2076	0.1699	5.10	5.20	4.49	5.85	4.49	5.86
10	9	3.85	4.02	0.2076	0.1889	5.11	5.15	3.26	4.56	3.25	4.55
10	10	3.85	3.85	0.2076	0.2076	5.12	5.12	2.47	2.47	2.43	2.43
15	1	3.25	10.00	0.2946	0.0230	5.79	8.05	5.98	7.36	5.97	7.38
15	2	3.25	7.5	0.2946	0.045	5.02	5.93	5.63	6.18	5.62	6.17
15	3	3.25	6.34	0.2946	0.0674	5.22	5.74	5.67	6.02	5.67	6.01
15	10	3.25	3.85	0.2946	0.2076	5.19	5.20	5.40	5.94	5.41	5.95
15	14	3.25	3.34	0.2946	0.2780	5.23	5.21	3.46	4.44	3.43	4.44
15	15	3.25	3.25	0.2946	0.2946	5.24	5.24	3.16	3.16	3.10	3.10
16	16	3.25	3.25	0.3109	0.3109	5.28	5.28	3.24	3.24	3.18	3.18
20	20	3.25	3.25	0.3721	0.3721	5.25	5.25	3.32	3.32	3.20	3.20
22	22	3.25	3.25	0.4007	0.4007	5.25	5.25	3.44	3.44	3.26	3.26
25	25	3.25	3.25	0.4411	0.4411	5.25	5.25	3.90	3.90	3.28	3.28
27	27	3.25	3.25	0.4665	0.4665	5.25	5.25	4.62	4.62	3.34	3.34
28	28	3.25	3.25	0.4787	0.4787	5.25	5.25	4.98	4.98	3.52	3.52
29	1	3.25	10.00	0.4907	0.0230	5.79	8.05	5.63	7.62	5.57	7.46
29	2	3.25	7.50	0.4907	0.0455	5.01	5.91	5.04	5.77	5.07	5.72
29	10	3.25	3.85	0.4907	0.2076	5.23	5.17	5.35	5.17	5.39	5.15
29	15	3.25	3.25	0.4907	0.2946	5.27	5.22	5.45	5.34	5.52	5.33
29	29	3.25	3.25	0.4907	0.4907	5.25	5.25	5.22	5.22	3.98	3.98
30	1	3.25	10.00	0.5024	0.0230	5.79	8.05	5.67	7.66	5.63	7.53
30	2	3.25	7.50	0.5024	0.0455	5.01	5.91	5.10	5.84	5.12	5.81
30	10	3.25	3.85	0.5024	0.2076	5.23	5.17	5.33	5.21	5.35	5.19
30	15	3.25	3.25	0.5024	0.2946	5.27	5.22	5.42	5.35	5.45	5.33
30	29	3.25	3.25	0.5024	0.4907	5.25	5.25	5.30	5.20	4.29	4.60
30	30	3.25	3.25	0.5024	0.5024	5.25	5.25	5.27	5.27	4.77	4.77

Notes: c_i, p_i, Δ_i are the marginal costs, equilibrium price and probability of forgetting for firm i. HHI^∞ is the expected long-run value of the HHI.

To investigate how far these results depend on the specific technology parameters, Figure D.1 shows the level of HHI^{32} under the social planner solution, and the dynamic equilibria when $\tau = 0$, 0.1, 0.25, 0.5 and 1 for the ranges of (ρ, δ) that we consider. When multiple equilibria exist, the maximum HHI^{32} is shown. The existence of multiple equilibria is the reason for the discontinuities in shading in the $\tau = 0$ and $\tau = 0.1$ figures. Figure D.2 shows the level of HHI^{32} under the social planner solution, and the unique equilibria when $\tau = 0$, 0.1, 0.25, 0.5 and 1 with static seller behavior (i.e., sellers' dynamic incentives are set to zero) in each state. Static and dynamic equilibria are identical when $\tau = 1$.

We observe that

- if $\tau = 0$, dynamic and static equilibrium HHI^{32} is generally below the level that the social planner would choose except for high ρ (limited LBD) and δs between 0.025 and 0.1. For this range of δs multiple equilibria are common, and the maximum equilibrium HHI^{32} is low but slightly larger than the social planner would choose.

- if $\tau = 0$, dynamic and static equilibrium HHI^{32}s are generally similar unless $\delta > 0.03$ (which implies $\Delta(m) > 0.37$ and $\Delta(M) > 0.6$ so depreciation rates are quite high). If $\delta > 0.03$, HHI^{32} tends to be larger in dynamic equilibria.

- dynamic equilibrium concentration generally increases sharply as τ increases from zero. Concentration when $\tau = 0.5$ is similar to concentration when $\tau = 1$, although there are examples where HHI^{32} is at a high level but declines slightly as τ increases from 0.5 to 1. Concentration increases more gradually with τ in the static equilibria.

- for $\delta < 0.03$, social planner concentration is somewhere between dynamic equilibrium concentration when $\tau = 0.25$ and dynamic equilibrium concentration when $\tau = 0.5$. This is consistent with our finding that TS^{PDV} is maximized for

τs around 0.3 for these δs (see text Figure 6(a)). For $\delta > 0.03$, concentration when $\tau = 0.5$ is also similar to the socially optimal levels.

The patterns observed for the illustrative technology parameter results are therefore fairly typical of what we see for other parameters that imply significant LBD effects if know-how depreciation is also limited. The next sub-section provides some analysis for $\rho = 0.95$ and $\delta = 0.03$, which is one example where $\tau = 0$ equilibrium concentration is above the social planner level.

Figure D.1: Expected Value of the HHI^{32} in the Social Planner Solution and Dynamic Equilibria. HHI^{32} value is the maximum across equilibria.

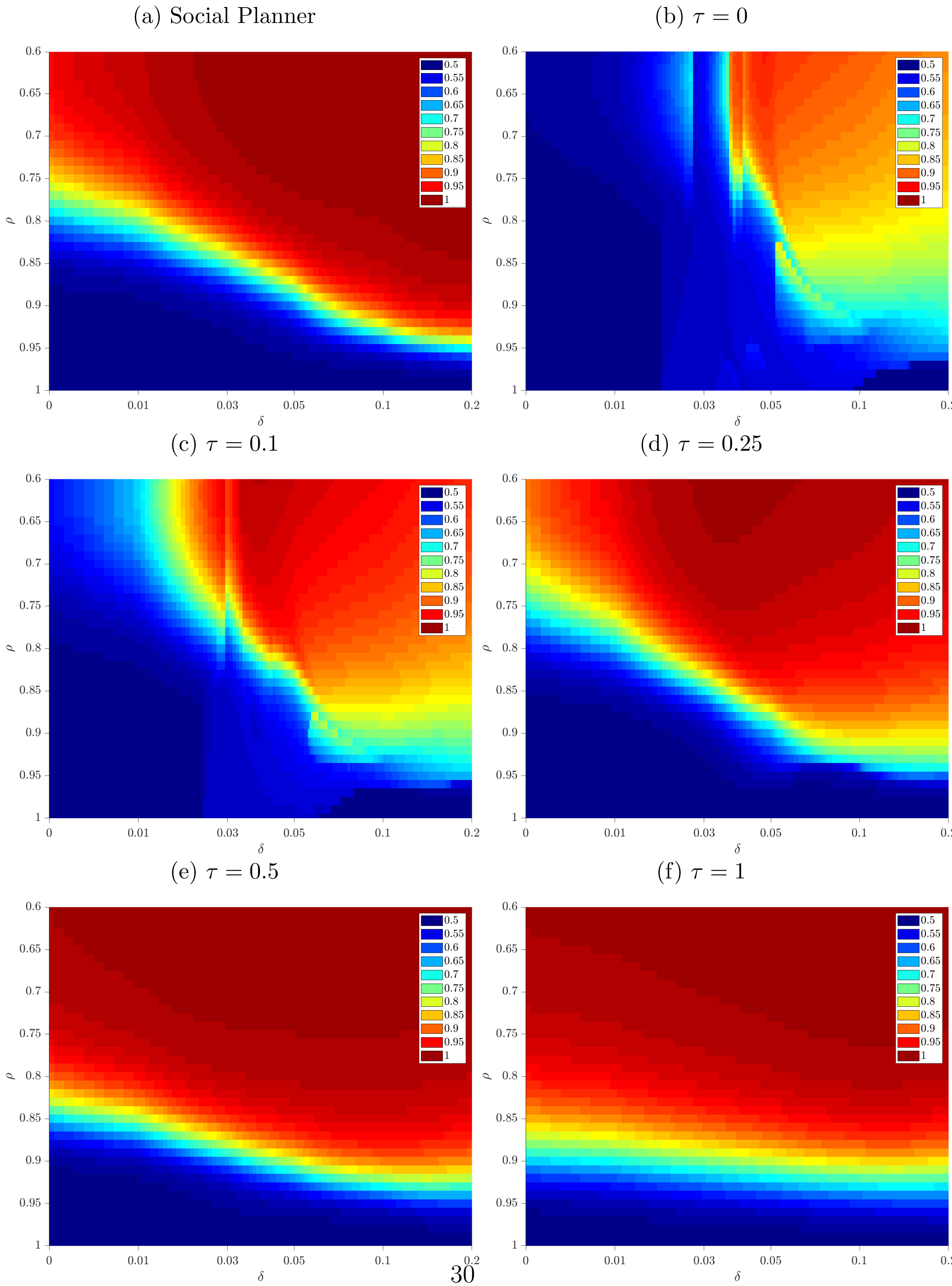


Figure D.2: Expected Value of the HHI^{32} in the Social Planner Solution and Equilibria with Static Seller Behavior. All static equilibria are unique.

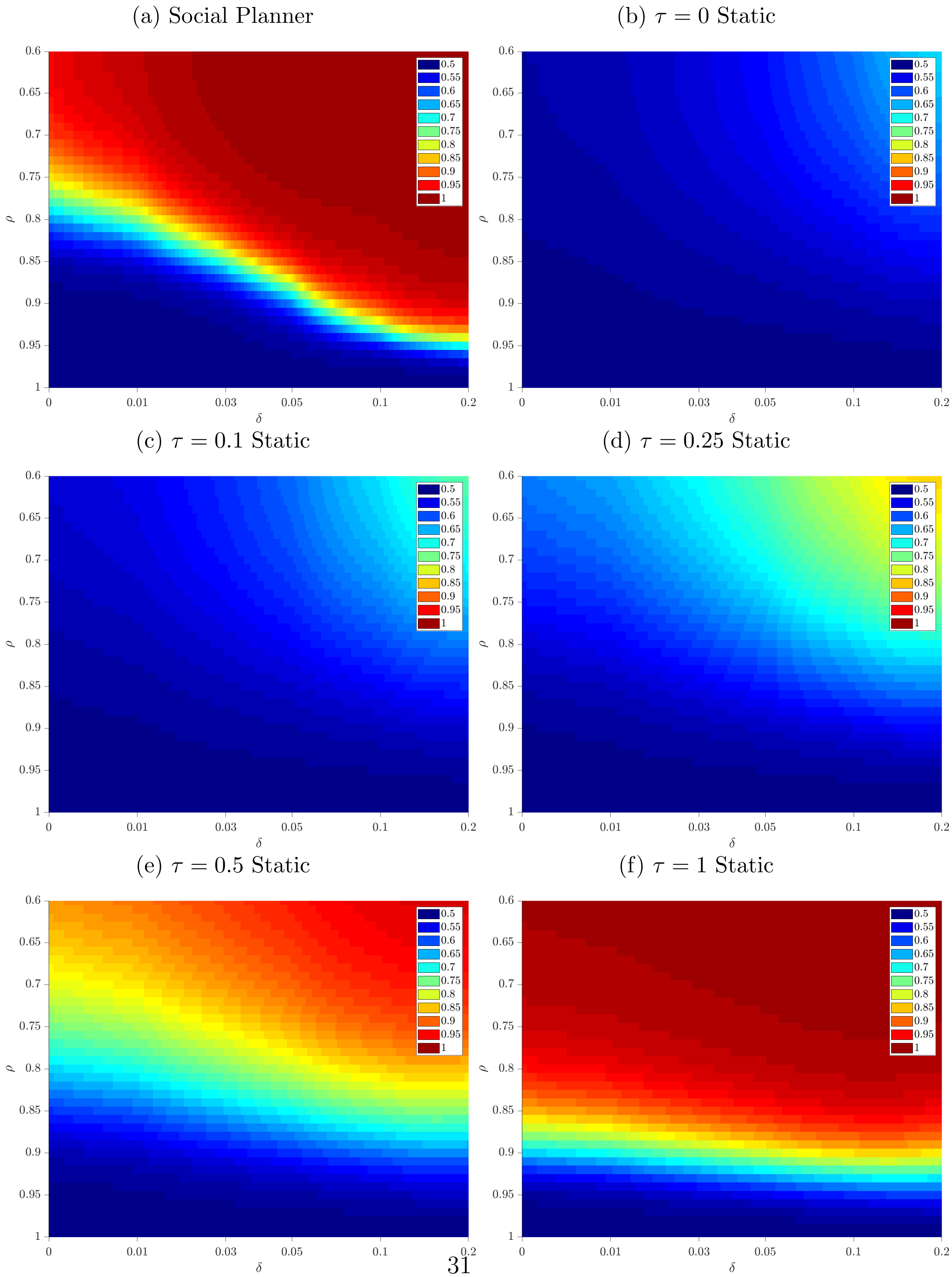
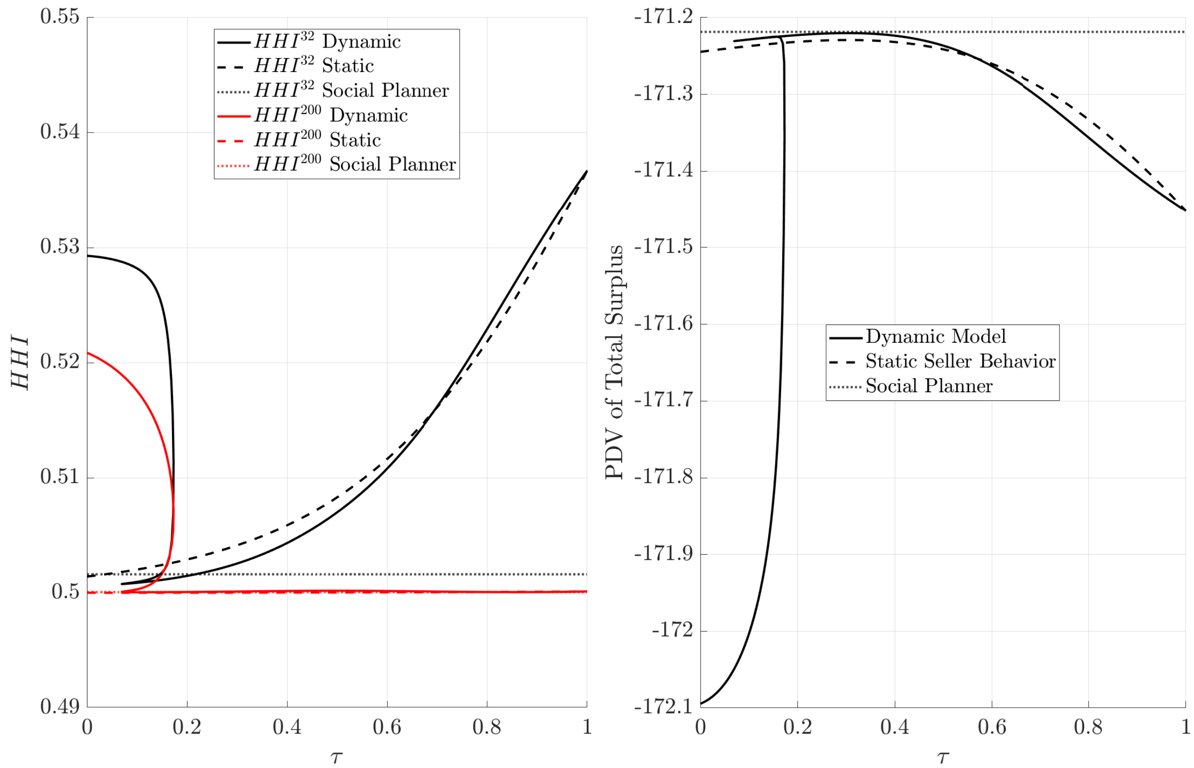


Figure D.3: The Effects of Changing the Allocation of Bargaining Power for $\rho = 0.95$ and $\delta = 0.03$ With No Policies.

(a) HHI^{32} and HHI^{200}

(b) TS^{PDV}



D.3 Effects of Bargaining Power For $\rho = 0.95$ and $\delta = 0.03$.

In the text, we use $\rho = 0.75$ and $\delta = 0.023$ as our illustrative technology parameters. For these parameters, market concentration is significantly lower than the social planner would choose when $\tau = 0$. In this subsection, we consider $\rho = 0.95$ and $\delta = 0.03$ as an example of technologies where $\tau = 0$ equilibrium concentration is higher than the social planner would choose. The discounted value of dynamic incentives also declines in τ and multiple equilibria exist for some low τ even though the $\tau = 0$ equilibrium is unique. $\rho = 0.95$ implies limited LBD: know-how can lower production costs by no more than 18%.

Figure D.3 replicates text Figure 7 for our new parameters. The unique equilibrium with $\tau = 0$ has concentration a little above the socially optimal level, and the level that would be generated by static seller pricing. The dynamic equilibrium τ-homotopy path bends back on itself so that there are multiple equilibria for $0.07 \leq \tau \leq 0.175$. In this range, concentration and welfare in the less concentrated equilibria are very close to the socially optimal and static equilibrium levels.[27] For $\tau \geq 0.175$, the equilibrium concentration increases with τ and efficiency declines for $\tau \geq 0.3$.

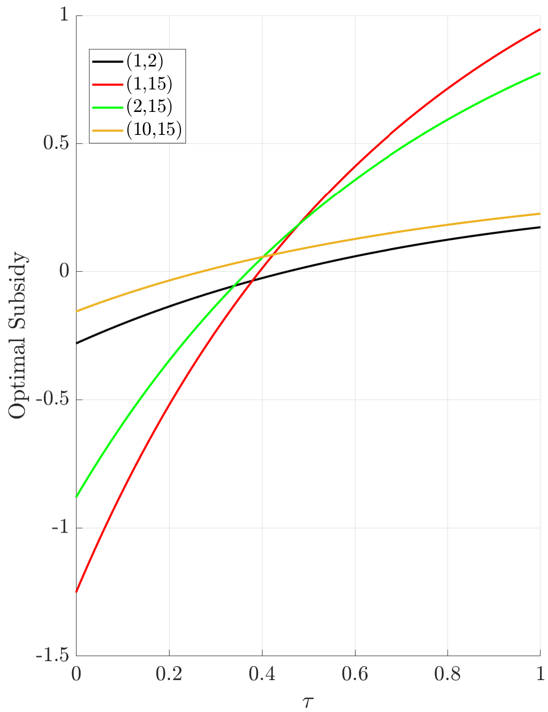
Policies. Figure D.4(a) shows the optimal subsidies that would implement the social planner solution as a function of τ. When $\tau = 0$, the subsidies are much smaller in scale than for the illustrative technology parameters, consistent with equilibrium concentration being closer to the socially optimal level. Even though equilibrium concentration is too high when $\tau = 0$, laggards are taxed when they make a sale. This reflects how, given socially optimal sales probabilities in other states, the laggard would be too likely to make a sale without a tax. Panel (b) shows that the $\tau = 0$ subsidy scheme lowers welfare if $\tau \geq 0.175$. For $0.08 \leq \tau \leq 0.175$ the scheme may increase or decrease welfare depending on which no-subsidy equilibrium is played. Therefore, the conclusion that subsidies that would maximize welfare when $\tau = 0$ can lower welfare for values of τ that are greater than zero but small remain.

Panels (c) and (d) show the effects of our stylized policies to promote competition. We only consider policies that are introduced from the start of the industry. As equilibrium concentration is very low until τ approaches 1, the concentration restriction has almost no effect on welfare or concentration until τ is large. The incentive policies increase welfare when $\tau \approx 0$, and they also increase welfare for $\tau > 0.7$.

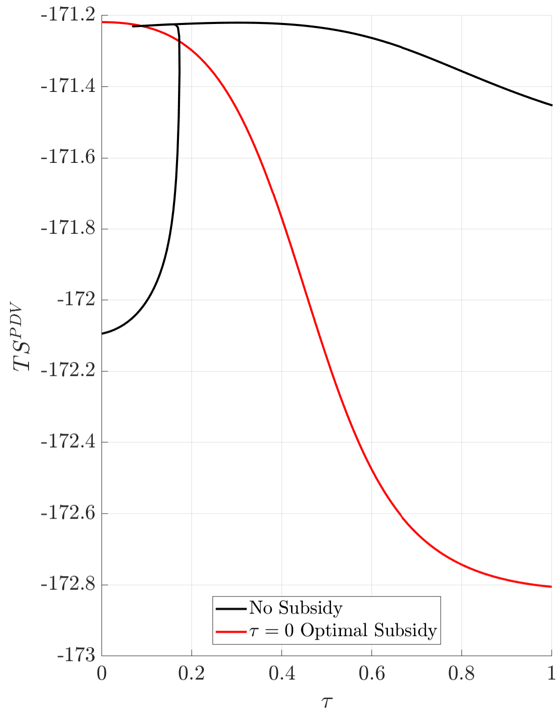
[27] The decline in concentration as τ increases is also associated with the leads of leaders tending to last for fewer periods, which also means that sellers' dynamic AB and AD incentives decline in τ, rather than increasing-then-decreasing as they do for the illustrative parameters.

Figure D.4: Policies and Bargaining Power for $\rho = 0.95$ and $\delta = 0.03$. Subsidies are given to the laggard when it makes a sale. This analysis assumes that policies are introduced at the start of the industry's life.

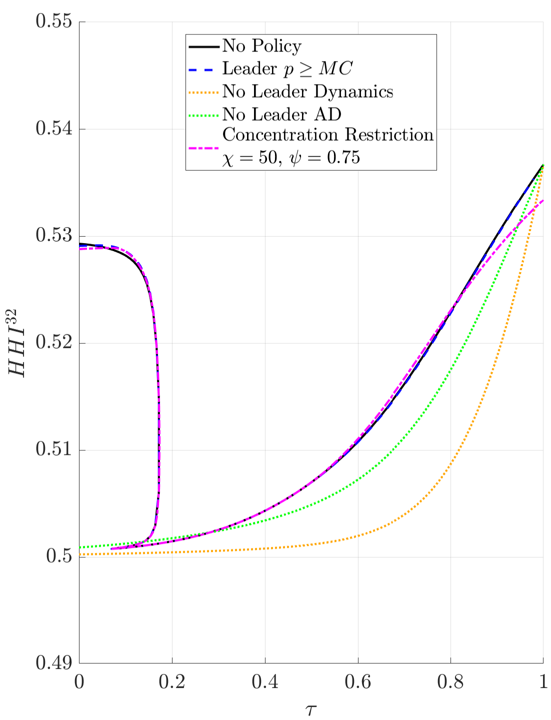
(a) Optimal Subsidies in Selected States



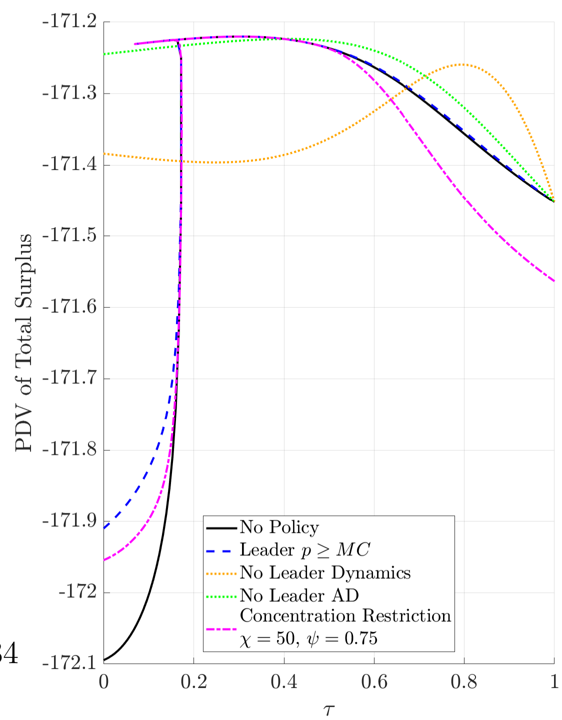
(b) Welfare and $\tau = 0$ Optimal Subsidies



(c) Policies Promoting Competition: HHI^{32}



(d) Policies Promoting Competition: TS^{PDV}



D.4 Alternative Concentration Restriction Policies.

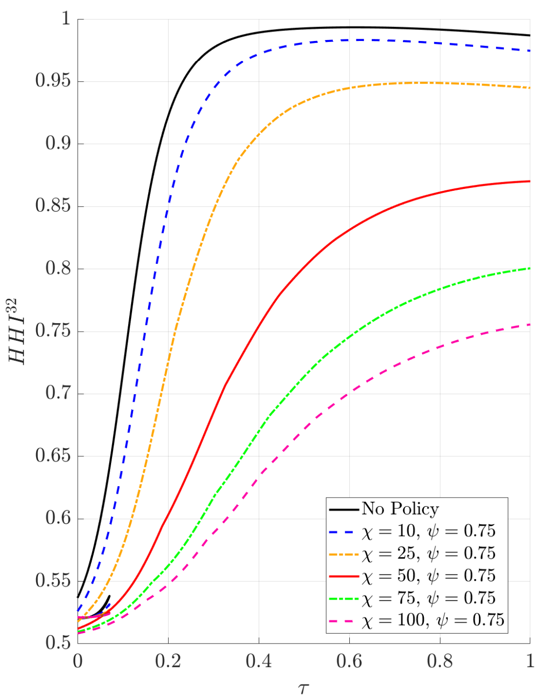
Text section 5.3 shows the effects of a concentration restriction policy, where the leader i has to pay a compliance penalty of $\chi \times \max\{0, D_i - \psi\}$ where $\chi = 50$ and $\psi = 0.75$. As $D_i = 0.5$ when firms are symmetric, and the maximum D_i approaches 1, $\psi = 0.75$ is a natural value to consider. We choose $\chi = 50$ as an example of a policy which lowers concentration but which still provides some probability that a firm will establish a know-how advantage that will lead to the compliance cost being incurred.

Figure D.5 shows how policies with different χs affect concentration and discounted total welfare (recall that we do not count the compliance cost as a total welfare loss) for the illustrative technology parameters, as a function of τ. Consistent with what one would expect, increases in χ lower concentration for values of τ where the share threshold would likely be breached with no policy in effect. For the values of χ that we consider, welfare falls as χ increases, although further analysis identifies that for $\tau \approx 0.4$ the policy can slightly increase TS^{PDV} when χ is slightly greater than zero.[28]

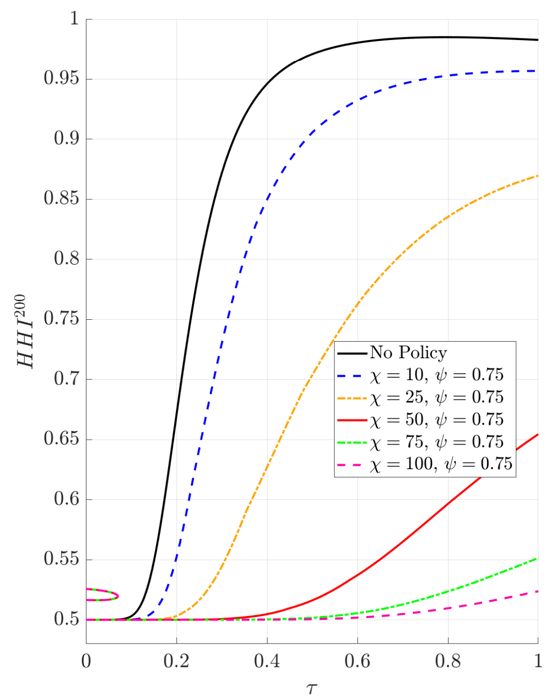
[28]For example, when $\tau = 0.4$, $TS^{PDV} = -90.4880$ when $\chi = 2$, compared to -90.4883 when $\chi = 0$ (no policy).

Figure D.5: Concentration and Welfare for Alternative Compliance Cost Parameters and the Concentration Restriction Policy, Assuming the Illustrative Technology Parameters.

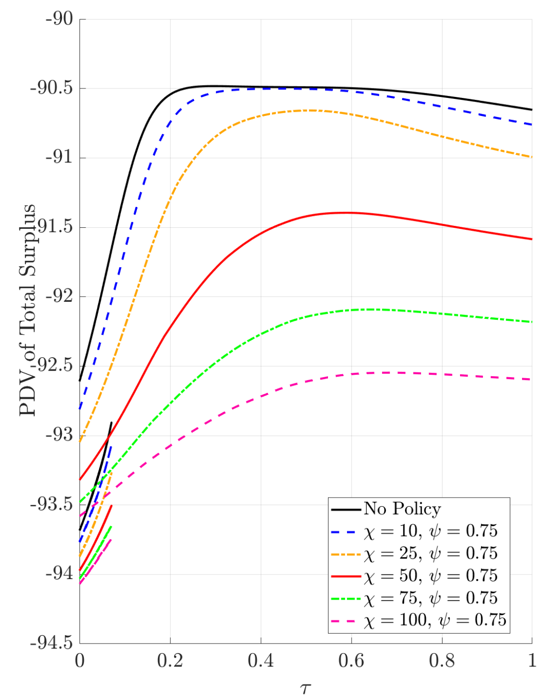
(a) HHI^{32}



(b) HHI^{200}



(c) TS^{PDV}



References

Besanko, D., U. Doraszelski, and Y. Kryukov (2014): "The Economics of Predation: What Drives Pricing When There is Learning-by-Doing?," *American Economic Review*, 104(3), 868–97.

Besanko, D., U. Doraszelski, Y. Kryukov, and M. Satterthwaite (2010): "Learning-by-Doing, Organizational Forgetting, and Industry Dynamics," *Econometrica*, 78(2), 453–508.